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Weaknesses in the fundamental processes among secondary school general mathematics pupils.

Boston University

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Thesis

WEAKNESSES IN THE FUNDAMENTAL PROCESSES AMONG SECONDARY SCHOOL GENERAL MATHEMATICS PUPILS

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CHAPTER 1.

THE PROBLEM

The problem of the weakness in fundamental processes of arithmetic among our high school general mathematics pupils is indeed one which needs much study as to its prevalence, its causes and the means of remedying the situation. It is not a new problem, one that we can blame "on the times." "In nearly all secondary schools there are at least a few pupils who have not learned the fundamental operations of arithmetic. If these pupils are ever going to make a successful adjustment to high school mathematics it is imperative that they be identified, their weaknesses diagnosed, and corrective treatment applied... All the kinds of tests mentioned--aptitude, achievement and diagnostic--are potentially useful in guidance."[1]

It is unfortunate that the weaknesses of our pupils are not recognized and corrected long before they arrive at the secondary school level. Yarbrough lists seven causes of failures in arithmetic, one of which is, "Teachers are failing to recognize, diagnose, and correct errors found in the work of their pupils as soon as they appear."[2]

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[1] Arthur E. Traxler; "A Testing Program for Mathematics at the Secondary School Level"; The Mathematics Teacher; Nov. 1946; p. 308-309

[2] Dorothy Yarbrough; Diagnosis of Pupils' Errors in Arithmetic with a View to Corrective Work Carried on through the Cooperation of the Teachers; unpublished Master's Thesis; Boston University School of Education; 1938; p. 108

If nothing is done about an error which occurs at the fifth grade level until the child is in the tenth grade, an incorrect habit has been made so strong that remedial work will be very difficult of accomplishment. Then too, a single error corrected early in the arithmetic history of the child will prevent the occurrence of many other related errors during his progress.

"Too often the small failing group in a class is overlooked, and the pace of the class work is set for the whole class by the brighter group."[1] As free public education is for all the children of all the people, the teacher should consider the needs of every pupil, not merely the majority. It is these weaker souls that really need and should have far more of the teachers' time and thought than is usually given to them. We need to search out their weaknesses, find the cause and rebuild, if necessary, their foundations so that their future progress will be happier, easier, and thereby more successful.

"Inadequate mastery of fundamental terminology, concepts and skills is probably the most outstanding cause for the difficulty encountered by individuals of all ages in dealing with anything of a mathematical nature."[2] It is with the

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[1] F. Lynwood Wren; "Secondary Mathematics"; Article in Encyclopedia of Educational Research; Walter S. Monroe, Editor; The MacMillan Co.; Boston, 1941; p. 707

[2] Ibid.; p. 707

fundamental skills that we are particularly concerned in this thesis and we see here that Wren feels that weakness in this field is responsible for difficulties in many later mathematical experiences. A point which cannot be stressed too strongly is the use of the word "mastery" in the above quotation. Nothing short of the goal of mastery will do when we are considering the ability to use the fundamental processes of arithmetic.

We often excuse errors in the work of a pupil by saying that he is careless, or that he really knows better. An erroneous assumption related to this thought is that we taught, therefore he must have learned. Speed tests often bring out this type of weakness and ought never to be used until we are sure that the goal of mastery has been achieved concerning the process with which we are working.

Our problem of weakness in the fundamental processes may well then be considered as incomplete mastery of the same. Anything short of mastery should be cause for concern on the part of the mathematics teacher as early in the experience of the child as it occurs.

This condition of weakness and the possibilities of remedying it was brought particularly to my awareness several years ago, when Practical Mathematics was a required subject for all tenth grade boys. One particular lad, a fine, courteous young man (and football star) failed the subject in the first quarter. His mother came in to see me and, in a

very pleasant way, asked that I give her son "a break" and pass him. She based her request on the fact that all through Junior High School he had failed in mathematics. In the course of our discussion it was brought out that he was absent from school for six months during the fifth grade with rheumatic heart trouble. Also brought to light was the fact that his major arithmetic difficulty was in the field of decimals. The boy was given "a break", though not exactly in the way that his mother expected. Each day a part of the mathematics period was spent by him in using that portion of a good workbook that dealt with decimals. I pointed out to both him and his mother that I felt I could conscientously give him a passing grade only if, in my judgment, he had made progress by doing this special work. He did progress very definitely and by the latter part of the year was able to compete creditably with the rest of the class in dealing with logarithms. This most certainly would not have been possible with his previous weakness in decimals.

The following year the subject was put on an elective basis and therefore some of the poorest mathematics students probably eliminated themselves from further labor in this field. However, it seemed advisable to put this "identification, diagnosis and corrective treatment" on a more scientific basis.

CHAPTER 2.

THE GROUP

The first question that arises is why these people have taken the particular subject and what do they hope the time spent will do for them. A few are girls, planning on entering the field of nursing. They are very definite in recognizing the need for strength in arithmetic computation and the elements of algebra. The majority of the students, however, are boys who will, nearly all, go to a trade school, apprentice school or directly into industry or the armed services. They are not taking mathematics for fun or because it is easy for them, but in practically all cases because they see its necessity and importance to them in their personal careers.

Page 6 gives the results of asking a group of forty-six Practical Math. I. students to record their answers to two questions which are pertinent at this point. First we ask, "Why did you take this subject?" and second, "What do you plan on doing after graduation from high school?" The pupils were told not to put their names on the paper so that as true a response as possible would be made. Only three elected the subject because they expected it to be easier than college mathematics. Two were forced into the class by parents who we may presume recognized the need and value for their children. Those listing college and engineering plans had had a difficult time with algebra and wished to get its fundamentals more strongly in mind before going on to their geometry.

Table 1. Reasons given by pupils in Pract. Math. I for taking the course (1952-1953)

Better foundation	11
Likes mathematics	7
Helpful later	6
Weak in algebra	5
Not ready for algebra	4
Refresher at 12th grade	3
Needed for nursing	3
Easier than college math	3
Parents' demand	2
Needed for Technical School	1
Did not pass geometry	1
	46

Table 2. Future plans of all pupils taking Practical Math. I, 1952-53

Trade Training School	11
Nursing	6
Military services	5
Business School	4
College	3
Engineering	3
Business	3
Police	2
Mechanic	3
Married	1
Aviation	1
Commercial artist	1
Electrician	1
Trade horses (Girl)	1
No answer	1
	46

"It is imperative for the teacher to be informed reliably concerning the ability of his students and of their mastery of the content of the subject being taught, if he is to adapt his instruction to their varying levels, correct their weaknesses, predict their success in advanced courses, and afford them guidance in accordance with their predispositions, attainments, and functioning interest."[1] A single test result or item of information is insufficient for the above purpose. We need to know each individual as well as possible and gather as much information as will enable us to see his strengths and also his weaknesses.

"Without some plan of discovering the particular defects of a pupil or a class, the work of the teacher is likely to be more or less futile."[2]

The first step in our plan is to secure as much information concerning the individual pupils as is already available in the records of the school. In many of our schools today this information is all to be found on the cumulative record card which contains all data collected from Grade I to Grade XII. This information and that which was subsequently secured as the program developed is all recorded in Table 3. on pages 8 to 17.

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[1] Arthur E. Traxler; "A Testing Program for Mathematics at the Secondary School Level"; The Mathematics Teacher. Nov. 1946. p. 308-309

[2] David E. Smith and William D. Reeve; The Teaching of Junior High School Mathematics; Ginn and Co., Boston, 1927; p. 330

Table 3

Cumulative Record Card Data and Subsequent Test Results of 91 Pupils

Pupil	Sex		Grade	Age	I.Q. (Otis)		Junior High Marks	Inventory Test		Inventory Retest	Diagnostic Test		Diagnostic Retest	Math. Review Test		Note: Decile 1 shall signify 0 decile through 1st decile; decile 2 shall signify from 1st decile through 2nd decile; decile 3 shall signify from decile 2 through decile 3; etc.
	Male	Female			Score	Decile		Score	Decile	Score	Score	%ile	Score	Score	Decile	
1	x		10	15-4	105	4	CLC	43	3	46	54	41	61			
2	x		10	15-2	106	5	CCB	44	3	49	62	73	61			
3	x		10	15-3	100	3	CLC	23	1	41	53	38	57			
4	x		10	15-5	102	4	LDD	40	2	40	50	30	55			
5	x		10	14-8	116	8	LCL	40	2	51	53	38	59			
6	x		10	16-7	85	1	CCC	44	3	50	63	78	59			
7	x		11	18-6	82	1	CLB	40	2	52	56	49	60			
8	x		11	18-1	84	1	LLD	44	3	53	60	65	51			
9	x		11	16-0	98	3	DDL	33	1	43	47	22	54			
10	x		11	17-0	105	4	LDC	41	2	50	62	73	60			

Table 3 continued

Pupil	Sex		Grade	Age	I.Q. (Otis)		Junior High Marks	Inventory Test		Inventory Retest	Diagnostic Test		Diagnostic Retest	Math. Review Test	
	Male	Female			Score	Decile		Score	Decile	Score	Score	Decile	Score	Score	Decile
11	x		11	16-11	89	1	LDC	34	1	47	46	20	54		
12	x		10	16-2	107	5	LLC	40	2	48	54	41	58		
13	x		10	15-6	105	4	-DL	38	2	50	55	45	62		
14	x		10	14-11	113	7	LCD	35	1	45	50	30	58		
15	x		10	15-9	103	4	-LL	39	2	44	53	38	50		
16	x		10	15-4	105	4	LLL	21	1	37	53	38	51		
17		x	10	15-11	112	7	LLE	42	3	51	55	45	58		
18		x	10	15-7	96	2	CCL	39	2	47	59	60	57		
19	x		10	14-7	109	6	-CC	36	1	51	39	9	51		
20	x		10	14-5	97	3	LLL	40	2	50	54	41	63		

Table 3 continued

Pupil	Sex		Grade	Age	I.Q. (Otis)		Junior High Marks	Inventory Test		Inventory Retest	Diagnostic Test		Diagnostic Retest	Math. Review Test		
	Male	Female			Score	Decile		Score	Decile	Score	Score	%ile	Score	Score	Decile	
21	x		10	16-0	88	1	DLD	40	2	40	52	35	58	23	2	
22	x		10	15-6	84	1	LDL	37	1	44	52	35	55	15	1	
23	x		10	16-4	104	4	CCL	37	1	49	63	78	57			
24	x		10	15-10	98	3	--B	38	2	44	58	56	49			
25	x		10	16-7	104	4	LCL	44	3	49	58	56	58			
26	x		11	16-1	98	3	CCB	41	2	50	60	65	65	39	9	
27	x		11	17-2	85	1	LLC	30	1	40	62	73	54	24	3	
28		x	11	15-8	110	6	LCD	34	1	47	52	35	56	19	1	
29	x		11	17-9	97	3	DDD	33	1	43	48	25	50	25	3	
30	x		11	18-7	93	2	DLC	15	1		41	11		22	2	Drop. Subj.

Table 3 continued

Pupil	Sex		Grade	Age	I.Q. (Otis)		Junior High Marks	Inventory Test		Inventory Retest	Diagnostic Test		Diagnostic Retest	Math. Review Test		
	Male	Female			Score	Decile		Score	Decile	Score	Score	%ile	Score	Score	Decile	
31		x	12	16-5	115	8	BLL	41	2		50	30		17	1	Left School
32	x		10	15-1	106	5	CCL	42	3	51	65	87	66	33	7	
33	x		10	16-8	90	1	CLL	38	2	50	59	60	61	20	1	
34	x		10	17-5	88	1	CDC	40	2	51	46	20	50	32	6	
35	x		11	17-1	105	4	DDC	35	1		57	52				Left School
36	x		11	16-10	105	4	DCL	39	2	51	64	84	62	38	9	
37	x		11	17-9	86	1	CCC	44	3	50	64	84	61			
38	x		11	16-4	104	4	LCB	38	2	48	62	73	60	32	6	
39		x	11	16-11	108	5	--B	44	3	48	40	10	53	20	1	
40		x	10	14-8	111	6	CCC	44	3		48	25		20	1	

Table 3 continued

Pupil	Sex		Grade	Age	I.Q. (Otis)		Junior High Marks	Inventory Test		Inventory Retest	Diagnostic Test		Diagnostic Retest	Math. Review Test		
	Male	Female			Score	Decile		Score	Decile	Score	Score	%ile	Score	Score	Decile	
41	x		10	14-9	91	2	CLL	42	3	51	48	25	38	35	7	
42	x		10	17-0	94	2	LLD	30	1		41	11				Left School
43	x		10	15-0	101	4	CCC	44	3	53	61	69	65	34	7	
44	x		10	15-4	93	2	DLC	40	2	52	53	38	57	23	2	
45	x		10	15-5	113	7	CCL	43	3	46	63	78	60	22	2	
46	x		10	18-11	87	1	DDC	20	1	39	40	10	39	21	2	
47	x		11	16-3	116	8	DDD	23	1	41	41	11	42	19	1	
48	x		10	15-5	103	4	CLC	41	2	48	53	38	59	22	2	
49		x	10	14-9	117	8	CCC	43	3	46	56	49	61	27	4	
50	x		10	15-2	84	1	LLD	41	2		41	11		16	1	

Table 3 continued

Pupil	Sex		Grade	Age	I. Q. (Otis)		Junior High Marks	Inventory Test		Inventory Retest	Diagnostic Test		Diagnostic Retest	Math. Review Test	
	Male	Female			Score	Decile		Score	Decile	Score	Score	%ile	Score	Score	Decile
51	x		10	15-5	92	2	ABC	42	3	52	55	45	63	23	2
52	x		10	14-6	101	4	LLC	44	3	48	59	60	52	31	5
53	x		10	15-3	121	9	CCC	37	1	48	49	88	56	38	8
54	x		10	14-10	112	7	CCC	43	3	51	65	87	66	41	9
55	x		10	15-11	90	1	CDL	36	1	45	55	45	57	19	1
56	x		10	14-3	90	1	--C	36	1	51	54	41	57	20	1
57	x		10	16-11	95	2	CCL	44	3	51	54	41	58	32	6
58		x	12	17-3	90	1	CCB	38	2	50	57	52	59	24	3
59	x		11	16-3	122	9	CLL	41	2	60	59	60		31	4
60	x		10	15-10	108	5	DDC	39	2	47	55	45	58	26	4

Table 3 continued

Pupil	Sex		Grade	Age	I.Q. (Otis)		Junior High Marks	Inventory Test		Inventory Retest	Diagnostic Test		Diagnostic Retest	Math. Review Test		
	Male	Female			Score	Decile		Score	Decile	Score	Score	%ile	Score	Score	Decile	
61	x		10	15-10	82	1	LLC	41	2	54	62	73	65	33	7	
62	x		10	14-11	95	2	CDD	43	3	49	59	60	57	30	6	
63	x		10	15-2	101	4	DDL	30	1	30	35	6	39	17	1	
64	x		10	15-9	82	1	LLD	42	3	42	43	14	46	27	5	
65		x	10	15-4	91	2	DDD	18	1							Dropped
66	x		10	16-8	112	7	CCL	42	3	54	57	60	63	29	5	
67	x		10	14-9	112	7	BCC	43	3	47	64	84	65	35	8	
68		x	10	17-8	78	1	CCL	31	1	45	43	14	46	12	1	
69	x		10	15-8	81	1	LCL	40	2	52	56	49	49	21	2	
70	x		10	14-9	103	4	LLL	38	2	54	57	52	46	15	1	

Table 3 continued

Pupil	Sex		Grade	Age	I.Q. (Otis)		Junior High Marks	Inventory Test		Inventory Retest	Diagnostic Test		Diagnostic Retest	Math. Review Test	
	Male	Female			Score	Decile		Score	Decile	Score	Score	%ile	Score	Score	Decile
71	x		10	15-11	108	5	LDL	28	1	39	41	11	45	16	1
72	x		11	15-9	113	7	LCC	44	3	52	52	35	63	19	1
73	x		12	17-0	109	6	BCL	44	3	49	61	69	65	28	5
74		x	12	16-6	114	7	LCL	41	2	52	55	45	57	33	7
75		x	12	18-0	94	2	DDD	43	3	51	45	18	56	23	3
76	x		10	14-10	114	7	LLL	40	2	49	57	52	61	35	8
77	x		10	15-8	99	3	LLL	40	2	49	52	35	56	33	7
78	x		10	15-5	97	3	CCL	43	3	51	56	49	62	29	5
79		x	10	16-9	77	1	DDD	31	1	44	49	28	50	13	1
80	x		10	17-3	76	1	-CL	29	1	29	50	30	50	19	1

Table 3 continued

Pupil	Sex		Grade	Age	I.Q. (Otis)		Junior High Marks	Inventory Test		Inventory Retest	Diagnostic Test		Diagnostic Retest	Math. Review Test	
	Male	Female			Score	Decile		Score	Decile	Score	Score	%ile	Score	Score	Decile
81	x		10	15-3	100	3	-CL	35	1	39	52	35	52	20	2
82	x		10	14-7	107	5	LDC	28	1	36	51	32	48	17	1
83	x		10	16-10	94	2	DCL	24	1	24	23	2	37	20	2
84	x		10	14-7	107	5	CLL	37	1	46	44	16	48	25	4
85	x		11	16-1	95	2	D--	34	1	47	45	18	54	20	2
86	x		11	15-11	93	2	BCC	38	2	49	52	35	60	25	4
87	x		11	15-9	104	4	LLD	44	3	50	54	41	60	25	4
88	x		11	15-8	111	6	CLC	44	3	52	48	25	56	26	4
89		x	12	18-0	90	1	LCD	35	1	46	43	14	56	26	4
90	x		12	17-1	98	3	LDD	27	1	46	51	32	57	26	4

91	Pupil	
x	Male	Sex
	Female	
12	Grade	
17-7	Age	
104	Score	I.Q. (Otis)
4	Decile	
BLD	Junior High Marks	
43	Score	Inventory Test
3	Decile	
49	Score	Inventory Retest
62	Score	Diagnostic Test
73	%ile	
61	Score	Diagnostic Retest
38	Score	Math. Review Test
9	Decile	

Table 3 continued

In the first column we find the sex of the pupil. Table 4 on page 18 gives the numbers of each sex in the special group of ninety-one pupils that this study covers. These pupils were those from all students taking Practical Math. I. whose scores on the Inventory Test, given at the start of the course, fell in the bottom thirty per cent.

Table 4. Distribution by Grade and Sex

	Grade X	Grade XI	Grade XIII	Total
Male	54	20	3	77
Female	7	2	5	14
Total	61	22	8	91

We cannot say from this result that sex is a determining factor in low mathematical ability, because 85% of this special group are boys, as compared to 87% of all Practical Math. classes that have been studied over a period of several years. Sex, then, need not be considered in any further study of this problem of ours.

The grade level of the students taking this course is tabulated next in Table 3. There are, as indicated in Table 4 on page 18, a very few 12th graders, a fairly large group at the 11th grade level, and a considerable majority are to be found in the 10th grade group. These numbers are significant only when we know the make-up of the entire group taking the subject. The 12th graders we may disregard, as they are very few in number in all of the classes under consideration. The 11th grade pupils who are in this special group are 32%

of all 11th graders taking Practical Math. I. Our deduction from this fact is that, after having terminated their mathematics at the end of the ninth year, they discovered for themselves that their serious weakness in arithmetic would be a real handicap to them later in life. They had no knowledge of what would be done about this weakness in the practical mathematics class, but their own awareness of a difficulty makes the treatment easier and satisfactory results more possible.

The next item that we find in Table 3. is the ages of the pupils under consideration. This age is as of September in the year that the subject is taken. Table 5. on page 20 shows graphically how the pupils in our special group under study compare with the average age for their particular grade level. At the grade ten level the average age in the school used in this study is 15 years 4 months. If we consider one year above or below this average as constituting a normal group, there are twelve pupils decidedly overage. In the grade eleven column we see five who are definitely beyond the average age for their grade. In the twelfth grade section we find none who are out of line seriously. There are two conclusions that may be drawn from this information that may begin to create a picture of our problem. First, some of these pupils may have started in school at a later age than the majority; and, second, ~~that~~ their reaction to the school experience was so poor that they repeated one or more grades

Table 5

Ages of Pupils by Grade Levels

Ages	Grade 10	Grade 11	Grade 12
18- 7		1	
18- 6		1	
18- 1		1	
18- 0			2
17-11			
17-10			
17- 9		2	
17- 8	1		
17- 7			1
17- 6			
17- 5	1		
17- 4			1 Median
17- 3	1		
17- 2		1	
17- 1		1	
17- 0	1	1	1
16-11	1	2	
16-10	1	1	
16- 9	1		
16- 8	2		
16- 7	2		
16- 6			1
16- 5			1
16- 4	1	1	
16- 3		2 Median	
16- 2	1		
16- 1		2	
16- 0	1	1	
15-11	3	1	
15-10	3		
15- 9	2	2	
15- 8	2	2	
15- 7	1		
15- 6	2		
15- 5	4		
15- 4	4 Median		
15- 3	3		
15- 2	3		
15- 1	1		
15- 0	1		
14-11	2		
14-10	2		
14- 9	4		
14- 8	2		
14- 7	3		
14- 6	1		
14- 5	1		

Note: The medians are for the entire school of approximately 2000 pupils

along the way.

Their weakness in mathematics at the present time may well be due to this poor reaction to the educational program and, if so, we need to reorient their thinking. Part of the success we may achieve in the remedial work that we contemplate depends upon a revised attitude towards mathematics in particular and probably school in its entirety. As they draw near to the time when they will leave school to go out into the world, they become increasingly aware of their mathematical needs and shortcomings. This awareness on their part makes the problem of remedial work far less difficult than it would otherwise be the case.

Another item that should be kept in mind when dealing with those who have missed out for so long is the matter of teacher personality. The teacher who leads the child patiently, quietly, yet with evident confidence can attain greater results than he might were he to depend on scientifically devised drill exercises alone. Sincerity on the part of the instructor is a very potent medicine with which to treat those with mathematical ills.

We now come in Table 3. to the column headed I.Q. Table 6. on page 22 gives the scores attained by a representative group of the school population consisting of 1070 mathematics students in both college and general curricula. The decile norms were based on this total group of 1070 cases.

Table 4

Intelligence Quotients (Otis) Grade 6 Level
(Latest available)

Intervals	Group I All Math Students Both College and General (1070)	Group II Group Having Weakness in Math. Ability (91)
142 - 145	7	
138 - 141	6	
134 - 137	12	
130 - 133	19	
126 - 129	35	
122 - 125	54	1
118 - 121	94	1
114 - 117	117	6
110 - 113	136	10
106 - 109	128 Median 108	10
102 - 105	116	15
98 - 101	103	10
94 - 97	86	10 Median 99
90 - 93	61	11
86 - 89	33	5
82 - 85	35	8
78 - 81	17	2
74 - 77	5	2
70 - 73	3	
66 - 69	3	

Range 67 - 145

Median 108

Cases 1070

Deciles - Group I

0	65.5
1st	90.2
2nd	96.8
3rd	101.0
4th	104.8
5th	108.3
6th	111.4
7th	114.8
8th	118.5
9th	124.0
10th	145.5

The scores are obtained from the Otis Quick Scoring Test of Mental Ability--Beta--Form B. as given at the sixth grade level, the latest on record in the system used for this study. The scores of the group used in our analysis have been placed to the right of those showing the total group range. We can see from this graphic representation that their range and median are not too serious by comparison with the total group. The general picture shown is that they are about ten points below the entire group. Nearly half of these folks are at the 100 I.Q. or better and this fact is most encouraging, since the I.Q. signifies, presumably, that these are reasonably educable. Probable success in a specific subject, mathematics, is not necessarily predictable by these I.Q. scores. In fact, the correlation between I.Q. and mathematics ability is decidedly low (-.19) as shown by the correlation chart Table 9. on page 29.

The last group of facts that we derive from the usual cumulative record card is the average mathematics marks in grades seven, eight and nine. These are summarized in Table 7. on page 24, and, in the case of most of the group, are the first facts upon which we can base our judgment of the need for remedial work to be done. Even these letter marks are certainly not to be used as a scientific basis for deciding who should be given remedial help. The As and Bs and probably many of the Cs would, in themselves, suggest that the pupil was in no particular need of remedial work. If those letter

marks were as objective as they should be in mathematics classes one could hardly expect to see such high marks when the later measured ability is found to be so low. Some teacher in the pupil's history must have been marking on goodness rather than achievement.

Table 7. Junior High Yearly Average Marks

Mark*	Grade VII	Grade VIII	Grade IX	Total
A	1			1
B	5	1	7	13
C	30	35	29	94
L	31	30	35	96
D	16	21	18	55

*A = 90-100
B = 80- 89
C = 70- 79
L = 65- 69
D = 0- 64

CHAPTER 3.

PROCEDURE

The information on the record cards is then of but small value to us in determining the degree with which an individual has mastered the fundamentals of computation. As an inventory is necessary in business establishments at the beginning of the new year, so also we need, with the beginning of secondary school mathematics, to ascertain as accurately as possible what each pupil has as a foundation with which to work. There are many standardized tests published from which a choice could be made for this purpose or the teacher might, with a proper amount of research, devise one for his own purposes. Great care needs to be taken in making a choice in order that the test will adequately measure so broad a field.

The aims of the course of study will be an important determinant in this selection. Since a major aim of the course, Practical Mathematics I., is preparation of the students for training in or work experience of industry, the test chosen for use at this point is rather unique. This test was compiled for use by the sheet metal industry during World War I in selecting young men for training in that branch of the war effort. It was printed in the October, 1922 issue of the Industrial Arts Magazine and appears, in the light of the test-retest correlation shown in Table 14 on page 52, to cover our needs very well. Mere examination of a test, however, is insufficient ground for determining its worth in a specific

situation. It needs to be used and the results appraised in the light of the group tested and the information gained.

The test (page 27) was given to all pupils taking Practical Math. I. during the first week of the course and the range, median and decile scores have been obtained from 905 students over a period of several years. The scores of the pupils under special study (Group B) are placed beside the range of Group A (Table 8). Each pupil in Group B can now be compared with the entire group by means of his decile score obtained.

As a means of determining what would come next, those pupils found to be in the lowest thirty per cent of all students taking the test were considered to be serious enough problems to warrant further testing and special remedial work. This position on the scale was taken arbitrarily and does not consider that many with scores above this point might profit by the program to follow. It would be difficult, however, to justify the time and work involved when there probably would be but little real increase in ability.

"Teaching success in mathematics depends very materially upon the effective use of diagnostic techniques and the complementary remedial procedures."[1] At this point we need to

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[1] F. Lynwood Wren; "Secondary Mathematics"; Article in Encyclopedia of Educational Research; Walter S. Monroe, Editor; The MacMillan Co., Boston, 1941; p. 707

MATHEMATICS INVENTORY TEST

TEST #1

1. ADD--607+396+275+609
2. 3759+6082+9603+7891
3. 6.572+9.371+6.205+7.864
4. 8.5+7.906+3.908+7.5
5. SUBTRACT--3841 from 9006
6. 57.36 " 79.21
7. 5.875 " 298
8. 2.4578 " 8.72
9. 16.9 " 697.2
10. 34.176 " 198.1
11. MULTIPLY--576 by 38
12. 894 " 262
13. 79.2 " 68
14. 62.4 " 1.45
15. 3.7226 " 2.008
16. DIVIDE----3927 " 3
17. 3746 " 5.1
18. 17226 " 783
19. 8.3895 " .235
20. 34.41348" .02631

TEST #2

1. ADD 1/8, 1/4, 3/4, 1/2
2. 3/16, 3/8, 5/32, 1/4
3. 1 1/4, 3 3/8, 2 1/64, 3 3/16
4. 1/4, 16 3/8, 2 3/4, 6 1/16
5. SUBTRACT 3/16 from 1/2
6. 5/16 " 3/4
7. 5/8 " 1 5/16
8. 1 3/4 " 27 1/16
9. 2 5/8 " 8 9/32
10. 7/16 " 4 3/64
11. MULTIPLY 1 7/8 " 1/8
12. 3 3/4 " 9/16
13. 5 3/8 " 2 3/8
14. 6 1/4 " 5 7/8
15. 5 3/4 " 8 1/4
16. DIVIDE .9225 by .375
17. 16 3/4 " 1/8
18. 34 1/16" 6 3/8
19. .39035 " .5275
20. 47 1/4 " 1.75

TEST #3

1. Change 10/16 of an inch to 8ths.
2. How many sixteenths in 3/4 in?
3. How many thirty-seconds in 3/4 inch?
4. Which is greater: 13/16 or 7/8 of an inch?
5. Which is greater: 3/10 or 3/12 of an inch?
6. Change to improper fractions:
   4 1/8
7. 7 1/2
8. 3 3/4
9. 2 1/6
10. 1 1/16
11. Express in whole or mixed numbers:
    21/16
12. 8/8
13. 24/3
14. 7/2
15. 121/12
16. A pound of alloy is made of 5 parts zinc, 2 parts tin, 1 part lead. What fraction of a pound of zinc, tin, lead is in the alloy?
17. A steel plate 2'6" wide is to be sheared into 4 strips of equal width. How wide will each strip be in inches?
18. Express in terms of lowest common denominator 5/16, 3/4, 9/32.
19. How many steel pins to be finished 1 1/2" long can be cut from a 7' rod if we allow 1/4" to be wasted in cutting each pin?
20. If an alloy is 67% copper to 33% zinc, how many pounds of each metal in a casting weighing 75 pounds?

Table 8

Mathematics Inventory Test Results and Norms

Intervals	Group A Scores all Students (905)	Group B Group with Math Fundamentals Weakness (91)	Retest at close of year. Group B
58 - 60	11		1
55 - 57	59		
52 - 54	125		10
49 - 51	177		22 Md. 50
46 - 48	189 Median 47		13
43 - 45	122	23	6
40 - 42	89	25 Md. 41	5
37 - 39	49	15	2
34 - 36	34	10	1
31 - 33	17	4	
28 - 30	13	5	1
25 - 27	7	1	
22 - 24	6	3	
19 - 21	4	3	1
16 - 18	1	1	
13 - 15	1	1	1
10 - 12			
7 - 9			
4 - 6	1		
0 - 3			

Range 6 - 60

Median 47

Cases 905

Deciles - Group A

0 -- 3.5
1st -- 37.4
2nd -- 41.7
3rd -- 44.2
4th -- 46.3
5th -- 47.6
6th -- 49.1
7th -- 50.1
8th -- 52.3
9th -- 54.0
10th -- 60.5

Table 9. Correlation Between I.Q. and Mathematics Inventory Test

I. Q. (Otis) \ Mathematics Inventory Test	15-17	18-20	21-23	24-26	27-29	30-32	33-35	36-38	39-41	42-45
76-80					1	2				
81-85						1		1	4	3
86-90		1					2	4	2	1
91-95	1	1		1		1	1	1	1	5
90-100			1		1		3	1	4	1
101-105			1			1	1	3	6	6
106-110					2		1	2	2	4
111-115							1		3	8
116-120			1						1	1
121-125								1	1	

Coefficient of Correlation = -.19

search out in more detail the particular weaknesses of each case. This search is facilitated by the use of one of the numerous diagnostic test forms to be found in guidance or testing laboratories. "It is the purpose of the diagnostic test to find out, if possible, why any special pupil is unable to succeed; and to reveal strengths as well as weaknesses. In addition to this the pupils assist the teacher greatly by locating not only their particular difficulties but the causes of these difficulties as well."[1] Here again, the teacher must select carefully and intelligently the best that is available in his own situation.

For this purpose in our study the Basic Skills in Arithmetic Test of Colorado State College of Education, published by Science Research Associates, was made available and gave the data necessary to progress more intelligently toward our goal of real improvement for all those who needed help so desperately. A copy of the test is inserted as pages 31 and 32 and is followed immediately by a copy of the diagnostic record sheet which graphically analyses the specific difficulties of each case.

"In our testing we must always remember that we are not examining grades, but individual children."[2] The time needed

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[1] David E. Smith and William D. Reeve; The Teaching of Junior High School Mathematics; Ginn and Co., 1927; p. 330

[2] S. C. Garrison and Florence Ryan; "Age-Grade-Sex Percentile Norms for Some Educational Tests"; Peabody Journal of Education; Vol. 1, No. 2; Sept. 1923

FORM **A** 31

TOTAL SCORE _____

PERCENTILE _____

Colorado State College of Education

BASIC SKILLS IN ARITHMETIC TEST

by

William L. Wrinkle, *Professor of Secondary Education*
Juanita Sanders, *Assistant Professor of Mathematics*
Elizabeth H. Kendel, *Associate Professor of Mathematics*

NAME **GRADE** **AGE** **DATE OF BIRTH** **BOY** **GIRL**

SCHOOL **CITY** **TEACHER** **DATE**

READ THIS PARAGRAPH CAREFULLY

This is a test of the simple skills in arithmetic which you and other people most frequently use. The purpose of the test is to find out how accurate you are in using these skills. Although you will have as much time as you need to finish this test, you should work as rapidly as you can without making mistakes, and you should not spend too much time on any one problem. The problems have been spaced so that you will have plenty of room to work out your answers on the test paper itself. After you have worked out the answer to each problem, record your answer in the Answer Column at the right of each page.

published by

SCIENCE RESEARCH ASSOCIATES

57 West Grand Avenue, Chicago 10, Illinois

Q-91-B

Please use number 7-371 when reordering this test booklet.

ANSWER COLUMN

(1) *Add*

7
0

(2) *Add*

8
9
2
6

(3) *Add*

36
98
65
18

(4) *Add*

479
894
273
686

(5) *Subtract*

763
464

(6) *Subtract*

4730
4041

(7) *Subtract*

3001
485

(8) *Multiply*

0
8

(9) *Multiply*

789
67

(10) *Multiply*

945
308

(11) *Multiply*

820
40

In answering the four division problems which follow, write the remainders (if there are any) as common fractions:

(12) *Divide* $4\overline{)0}$

(13) *Divide* $3\overline{)2830}$

(14) *Divide* $96\overline{)76801}$

(15) *Divide* $42\overline{)8526}$

Change each of the following to its simplest form:

(16) $\frac{6}{10} =$

(17) $3\frac{4}{8} =$

(18) $14\frac{3}{3} =$

(19) *Change to a mixed number:* $\frac{19}{8} =$

(20) $2\frac{3}{4} = \frac{\quad}{4}$

(21) $\frac{2}{3} = \frac{\quad}{6}$

(22) 7 is what part of 28?

(23) *Add*

$39\ 5\frac{7}{12}$

(24) *Add*

$15\frac{3}{4}$
$12\frac{1}{2}$

(25) *Add*

$\frac{3}{4}$
$4\frac{1}{3}$
$5\frac{5}{6}$

(26) *Subtract*

$\frac{1}{2}$
$\frac{1}{3}$

(27) *Subtract*

$15\ 6\frac{3}{4}$

(28) *Subtract*

$23\frac{1}{4}$
$8\frac{2}{3}$

For items 29 through 36, each answer should be written in its simplest form.

(29) $\frac{1}{2}$ of $\frac{1}{3} =$

(30) $24 \times \frac{3}{4} =$

(31) $\frac{2}{3} \times 5 =$

(32) $15 \times 3\frac{2}{3} =$

(33) $2\frac{1}{2} \times 3\frac{3}{4} =$

(34) $\frac{1}{2} \div 2 =$

(35) $\frac{4}{5} \div \frac{4}{5} =$

(36) $6 \div \frac{2}{3} =$

(37) Write 15 thousandths as a decimal. _____

(38) Which of the following is the way you would read .06?
(six-tenths) (six) (six-thousandths) (six-hundredths) _____

Rearrange the numbers in the next two problems so that the largest number is first and the smallest is last.

(39) 7 .007 .08 _____ (largest) _____ (middle) _____ (smallest)

(40) .043 .23 .3 _____ (largest) _____ (middle) _____ (smallest)

(41) Change .9 to a common fraction. _____

(42) Change $\frac{1}{9}$ to a decimal. *(Leave the remainder as a common fraction after the second decimal place.)* _____

(43) $7 + .8 + .9 =$ _____

(44) $\$5.76 + \$25.90 + \$.49 = \$$ _____

(45) $\$73.80 - \$7\ 17 = \$$ _____

(46) Take \$8.11 from \$10.00. The remainder is \$ _____

(47) $\$5.47 \times 10 = \$$ _____

(48) $.35 \div 10 =$ _____

Place the decimal point correctly in the answers to the following problems. Add any zeros that may be needed.

(49) $.06 \times .9 = 54$

(50) $\$3.75 \times .6 = \$2250$

(51) $25\overline{)9.25}$ — 75, 175, 175, 000

(52) $25\overline{).925}$ — 75, 175, 175, 000

(53) $.25\overline{)925}$ — 75, 175, 175, 000

(54) $2.5\overline{)9.25}$ — 75, 175, 175, 000

Change the following decimal numbers to per cents:

(55) .04 = _____ %

(56) .025 = _____ %

(57) 1.2 = _____ %

ANSWER COLUMN

(58) Write 8% as a decimal. _____ (58) _____

(59) Write $\frac{1}{2}$% as a decimal. _____ (59) _____

(60) Write 105% as a decimal. _____ (60) _____

(61) Change $\frac{1}{3}$ to per cent. _____ % (61) _____ %

(62) Change 40% to a common fraction. _____ (62) _____

(63) How much is 2% of \$3.50? \$_____ (63) \$ _____

(64) How much is $12\frac{1}{2}$% of \$160? \$_____ (64) \$ _____

(65) How much is 150% of \$50? \$_____ (65) \$ _____

(66) \$5 is what per cent of \$50? _____ % (66) _____ %

(67) What per cent of \$100 is \$300? _____ % (67) _____ %

(68) Harry Jackson loaned \$500 to James Smith for six months. He charged Mr Smith interest at the rate of 6% per year How much was the interest on the loan? \$_____ (68) \$ _____

DIAGNOSTIC CLASS RECORD SHEET

for use with the

Basic Skills in Arithmetic Test

by W. L. Wrinkle, J. Sanders and E. H Kendel

Colorado State College of Education

This diagnostic record sheet is designed to help the teacher identify the specific difficulties encountered by each individual student and also by the class as a whole in the use of the fundamental arithmetical skills measured by this test.

After scoring the tests, enter each student's name, raw score, and corresponding percentile rank on the record sheet. Then, indicate in the proper columns the errors and omissions made by each pupil. The basic skill represented by each test item is identified at the top of the column, for example, problem 1 is an addition problem with zero difficulty, and problems 24 and 25 are addition problems involving unlike fractions.

The tabulation of each student's errors will provide a basis for appropriately directed individualized instruction. By totalling (vertically) the number of errors made by the entire class in each type of problem, the teacher will be able to determine which basic skills might profitably be given attention by the class before work is begun on more advanced skills.

Published by

SCIENCE RESEARCH ASSOCIATES

57 West Grand Avenue, Chicago 10, Illinois

Please use number 7-377 when reordering this class record sheet.

Form _____

Basic Skills in Arithmetic Test

Grade _____ Date of Test _____

Teacher _____

School _____

STUDENT'S NAME

RAW SCORE

PERCENTILE RANK

WHOLE NUMBERS

Addition

1—Zero difficulty

2—Single column

3-4—Carrying

Subtraction

5—Borrowing

6-7—Borrowing and zero difficulty

Multiplication

8—Zero difficulty

9—Carrying

10-11—Carrying and zero difficulty

Division

12—Zero difficulty

13—Carrying; one digit divisor; no zeros in quotient

14-15—Carrying; two digit divisor; zero in quotient

FRACTIONS

16-17-18—Changing a fraction to its simplest form

19—Changing an improper fraction to a mixed number

20—Changing a mixed number to an improper fraction

21—Changing a fraction to an equivalent fraction

22—Finding what part one number is of another

23—Adding whole and mixed numbers

24-25—Adding unlike fractions

26—Subtracting unlike fractions

27—Subtracting a mixed number from a

28—Subtracting mixed numbers

29—Multiplying fractions

30—Multiplying a whole number by a fraction

31—Multiplying a fraction by a whole number

32—Multiplying a whole number by a mixed number

33—Multiplying mixed numbers

34—Dividing a fraction by a whole number

35—Dividing fractions

36—Dividing a whole number by a fraction

DECIMALS

37—Writing decimals

38—Reading decimals

39-40—Comparing decimals

41—Changing a decimal to a common fraction

42—Changing a common fraction to a decimal

43—Adding decimal fractions

44—Adding a decimal mixed fraction and a decimal fraction

45—Subtracting decimal mixed numbers

46—Subtracting a decimal mixed number from a whole number

47—Multiplying a decimal mixed number by a whole number

48—Dividing a decimal fraction by a whole number

49-50—Placing the decimal point correctly in multiplying

51-52-53-54—Placing the decimal point correctly in division

PER CENTS

55-56—Changing a decimal fraction to per cent

57—Changing a decimal mixed number to per cent

58-59-60—Changing per cents to decimals

61—Changing a common fraction to per cent

62—Changing per cent to a common fraction

63-64-65—Finding the per cent of a number

66-67—Finding what per cent one number is of another

68—Computing simple interest

1

5

10

15

20

25

30

TOTAL

Class Distribution Table

Total Number Right	Tallies	Frequency
68		
64-67		
60-63		
56-59		
52-55		
48-51		
44-47		
40-43		
36-39		
32-35		
28-31		
24-27		
20-23		
16-19		
12-15		
8-11		
4-7		
0-3		
Number of Cases		
Median		

Directions for Distributing Scores and Obtaining Median

The median [1] is the mid-point in a series in which all scores have been arranged in order of size. It may be described as that point above and below which lie one half or 50 per cent of the scores or frequencies. The median may be computed by the following steps:

1. Fold page on dotted line so that the RAW SCORE column on the inside page appears to the left of the CLASS DISTRIBUTION TABLE.

2. Let us assume that student number one made a raw score of 46 on the test. Since his score falls within the interval 44-47, you would put a tally mark on this line in the "Tallies" column of the CLASS DISTRIBUTION TABLE.

3. Suppose that student number two made a raw score of 59. Then you would put a mark in the "Tallies" column to the right of the interval 56-59, since his score falls within that interval.

4. If the next pupil's score was 44, it, too, would be recorded to the right of the interval 44-47, along with that of pupil number one.

5. Use this procedure for distributing the scores of all the pupils in your class. Then, add the tallies for each interval horizontally, recording the total in the "Frequency" column. This shows you the number of students in your class who made scores of 68, 64-67, 60-63, etc. At this point, you may obtain a rapid check upon the accuracy of your work by totaling vertically the numbers in the "Frequency" column. Your total should equal the number of students in your class who took the test.

6. Now you are ready to compute the median for your group. Follow these directions carefully:

   a. Since one half of the scores fall below the median, we must determine how many scores constitute 50 per cent of the total number of scores in the distribution. If you have 27 scores, half of the number would be 13.5.

   b. Now we must determine below what point in the scale 13.5 scores will lie. Counting from the foot of your frequency column, total the frequencies, beginning with the lowest interval, up to but not including the interval which contains the middle case (13.5). Thus, if you have a cumulated frequency of 12 from the interval 0-3 through 36-39, and there are 5 students who received scores in the interval 40-43, you know that the median will be at least 40 and will fall within the interval 40-43.

   c. Subtract the cumulated frequency from half the number of scores (i.e., 13.5−12=1.5). 1.5 frequencies represent $\frac{1.5}{5}$ of the total number of scores within the interval. Since the range of the interval is 4 units, the point desired is $\frac{1.5}{5} \times 4 = 1.2$ score units above the lower limit of the interval.

   d. Finally, add to the lower limit of the middle interval group the quotient obtained in the preceding step (40+1.2=41.2). This number, 41.2, is the median for the class.

The median is called an average of position because it is affected by the number of scores and not by extremely high or extremely low scores. It can be used to advantage in describing a class or in comparing one class with another. Each student's score may be compared with the median to determine whether his achievement is higher or lower than the class average; however, the percentile norms given in the manual will be more useful in interpreting individual scores.

[1] *More detailed information concerning the computation of medians and the analysis of test scores may be found in TESTING AND COUNSELING IN THE HIGH SCHOOL GUIDANCE PROGRAM, by John G. Darley, Science Research Associates, 1943.*

to discuss results of tests with each student in question is well spent. In a class we can never know surely whether the individual is reacting favorably or otherwise to our discussions.

Before discussing the results of the inventory test and administering the diagnostic test it becomes necessary to set the stage for the acceptance of this information and its implications by not only the individuals concerned, but also the entire class. The first point to be put over is that it is no disgrace to be weak, but it certainly is one to fail to use any means to overcome this weakness if possible. I find that with this approach, the ones doing remedial work show no feeling of inferiority and are not looked down upon by the rest of the class. Various possible reasons for the failure of the individuals to acquire a reasonable facility in the use of the fundamentals of arithmetic should be discussed at this time also. This discussion will help to dispel any possible attitude of inferiority on the part of the ones who are going to undertake this new approach to the mastery of the subject. Some possible suggestions that could be made are: first, a prolonged absence at some period in the elementary school history due to serious illness or a major operation. Another suggested reason is the matter of moving about from one school system at various times. This was particularly prevalent with the families of military personnel during the war. The possibility that the pupil may not have

had a teacher whose personality was in harmony with his own cannot be overlooked. With considerable tact, we might even present the thought that some teacher along the way was not particularly well prepared either from the standpoint of subject matter or of educational practice.

The importance of this part of the procedure may be summed up in the following quotation. "The development of proper attitudes toward one's work is as important as is the acquisition of knowledge itself."[1] Selection of material and its administration is a mechanical process, often based on formulae. Influencing the attitudes of one or a group of individuals is often dependent upon the personality and/or the evident sincerity of the teacher. The pupil can always recognize whether we are putting on an act or are sincerely interested in his personal welfare. Remedial work, to be effective, requires that the teacher be well founded in the mechanics of the work, but also that his code of ethics in teacher-pupil relationship be well considered and of the highest order.

The Diagnostic Record Sheet shows, by means of checking the errors, the main division in which the pupil is deficient. The three divisions, fractions, decimals and per cent are the ones with which we shall be particularly concerned. Whole

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[1] Anthony, Kate, et.al.; "The Development of Proper Attitudes Toward School Work"; School and Society; 2:926, Dec. 1915.

numbers would be a question with only the most unusual person at the high school level. After recording the errors for each pupil, it will be very obvious in which of these three fields his major difficulty lies. For the group in this study, the distribution of cases within these three fields was: Fractions, 22; Decimals, 11; and Per cent, 58.

"Carefully planned review and drill contribute to the efficiency of any instructional program."[1]

"Drill only those pupils needing drill and check constantly to see if drill is producing accurate results."[2]

"Accuracy and facility in numerical computation are of such vital importance, however, to every individual that effective drill in the subject should be continued throughout the secondary school period, not in general as a separate topic."[3]

For these people whom we have discovered to be in need of drill work in one of the fields of arithmetic shown above, a choice of material is now necessary that will produce the most

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[1] F. Lynwood Wren; "Secondary Mathematics"; Article in Encyclopedia of Educational Research; Walter S. Monroe, Editor; The MacMillan Co., Boston, 1941; p. 708

[2] C. N. Kendall and G. A. Mirick; How to Teach the Fundamental Subjects; Houghton Mifflin, Boston, 1915

[3] John Wesley Young; Editor; The Reorganization of Mathematics in Secondary Education; A Report by the National Committee on Mathematical Requirements under the Auspices of the Mathematical Association of America Inc., Houghton Mifflin Co., 1927; p. 7

Table 10

Basic Skills in Arithmetic Test
Test-Retest Results

Basic Skills in Arithmetic Diagnostic Test at the Beginning of the Program		Basic Skills in Arithmetic Retest at the Half Year	
Intervals	Frequency	Frequency	
66 - 68		2	
63 - 65	8	9	
60 - 62	10	16	
57 - 59	12	20	Median 58
54 - 56	16 Median 56	12	
51 - 53	15	6	
48 - 50	10	9	
45 - 47	5	4	
42 - 44	4	1	
39 - 41	8	2	
36 - 38		2	
33 - 35	1		
30 - 32			
27 - 29			
24 - 26			
21 - 23	1		
18 - 20			

Range 13 - 68

Median 56

Cases 1351

These Range, Median and Cases are those of the publishers of the test.

Table 11. Correlation Between Inventory Test and Basic Skills in Arithmetic Test

Inventory Test	Basic Skills in Arithmetic Test									
	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	61-65
15-17						1				
18-20					1					
21-23						1		2		
24-26		1								
27-29						1	1	2		
30-32				1		2	1			1
33-35						2	4	2		1
36-38					1	1	1	5	4	2
39-41						1	3	10	6	3
42-45					1	2	3	6	7	10

Coefficient of Correlation .73

satisfactory treatment for the disability. "One important development growing out of the testing movement has been the creation of practice exercises which aim to remedy the weaknesses disclosed by the standardized tests."[1]

As there will be three fields of fundamental arithmetic being taught at the same time that the class as a whole is working on advanced mathematics, the remedial material used will need to be self-instructional in nature. There is considerable that is published in the form of workbooks that should produce satisfactory results, but here again sufficient care in selection will insure the best results.

Arithmetic Workbook 7 by Knight, Studebaker and Tate and published by Scott, Foresman and Company was available within the department and gave indication of fitting the situation at hand very well. There are short drills on specific processes, logically arranged in order of difficulty. Reference is made with each exercise to where the corresponding Self-Help instructional material will be found. The mechanics of using the book can be easily explained to the pupils and once understood they are on their own from there out.

This special work is not extra in any way but is done at the same time that the balance of the class is at another

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[1] John Wesley Young; Editor; The Reorganization of Mathematics in Secondary Education; A Report by the National Committee on Mathematical Requirements under the Auspices of the Mathematical Association of America Inc., Houghton Mifflin Co., 1927; p. 136

task. Each exercise is scored and the grades made equivalent to those earned by the regular students during this part of the class period. The scores earned on this work are a definite part of the quarterly grades. It is explained to the students that it is progress in mathematics that determines their marks.

We have discovered a weakness, isolated it specifically and given a treatment. Some might say that the problem has been satisfactorily solved at this point. However, further testing is necessary in order to ascertain if progress is being made and to encourage the pupil himself with evident improvement. It is too long to wait until the end of the year to see if the treatment is working. The pupil might well lose his incentive and even regress in his ability to handle the fundamental processes. "If the proper kinds of tests are given, the progress made by pupils can be measured, and the norms of accomplishment thus established can be used as a guide in discovering the educational needs of future classes."[1]

At the end of the first ten week period of the year appears to be a reasonable point at which we should be able to see some real advancement in many of the cases. At this time we could repeat the inventory test which was given at the start of the year or work out carefully an equivalent test for

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[1] David E. Smith and William David Reeve; The Teaching of Junior High School Mathematics; Ginn and Co. 1927, p. 330

use at this time. A test which is entirely different in construction and method of procedure tends to be more interesting and challenging to the pupils. It must, however, measure the same concepts as the earlier one. The Review Test on Fundamental Processes on pages 43-46 gave both satisfactory and encouraging results. The chart in Table 12 on page 47 shows the results for all students taking the subject as well as the results for our special students under treatment. The deciles are based on all pupils (170) taking Practical Math I. It is graphically obvious from this table that our special group more nearly distributes itself in keeping with the total group tested than it did at the start of the year on the inventory test. No chart identical with this one in Table 12 is available, but if we recall that these ninety people were all at or below the third decile earlier, their comparison with the group as a whole is most encouraging. An enlarged copy of this Table 12 may well, with a modicum of explanation, be shown to the class and both encourage those on treatment and possibly surprise the entire group. The results at this period are encouraging to the individual pupils, and should act as an incentive to further drill with the hope of gaining yet stronger basic mathematical abilities.

At the close of the half year period is another point where a check on progress is advisable. Readministering the Basic Skills in Arithmetic Test will show the pupil how far he, individually, has progressed on this scientifically

REVIEW TEST ON ARITHMETIC FUNDAMENTALS

Directions: Place an X in the circle which is below the letter that refers to the smallest number or value in each of the following groups.

1.	A 2/3	B 3/4	C 1/2	D 5/6	A	B	C	D
					O	O	O	O
2.	E 1/3	F 1/4	G 1/2	H 5/12	E	F	G	H
					O	O	O	O
3.	A 15/16	B 3/4	C 9/16	D 7/8	A	B	C	D
					O	O	O	O
4.	E 7/4	F 1 1/4	G 13/4	H 2 3/4	E	F	G	H
					O	O	O	O
5.	A 17/48	B 2/24	C 7/16	D 13/32	A	B	C	D
					O	O	O	O
6.	E .32	F .3	G .27	H 3.2	E	F	G	H
					O	O	O	O
7.	A 1.3	B .19	C .185	D .2	A	B	C	D
					O	O	O	O
8.	E .008	F .78	G .064	H 8.6	E	F	G	H
					O	O	O	O
9.	A 1.003	B .301	C .013	D .103	A	B	C	D
					O	O	O	O
10.	E .094	F .409	G .049	H .904	E	F	G	H
					O	O	O	O
11.	A LXI	B XLI	C XLVI	D LIV	A	B	C	D
					O	O	O	O

12.	E	MD	F	MM	G	MC	H	MCM	E	F	G	H
									O	O	O	O
13.	A	XIX	B	XVIII	C	XX	D	V	A	B	C	D
									O	O	O	O
14.	L		F	XLIX	G	C	H	LX	E	F	G	H
									O	O	O	O
15.	A	CD	B	DC	C	M	D	CCC	A	B	C	D
									O	O	O	O
16.	E	1/8%	F	12 1/2%	G	8%	H	18%	E	F	G	H
									O	O	O	O
17.	A	75%	B	3/4%	C	7.5%	D	07%	A	B	C	D
									O	O	O	O
18.	E	3 3/4%	F	375%	G	.375%	H	37 1/2%	E	F	G	H
									O	O	O	O
19.	A	126%	B	5/4%	C	1 1/4%	D	1.2%	A	B	C	D
									O	O	O	O
20	E	5/8%	F	62.5%	G	6.20%	H	60%	E	F	G	H
									O	O	O	O
21	A	1/3	B	.2	C	1/4	D	.15	A	B	C	D
									O	O	O	O
22	E	.30	F	.275	G	3/16	H	1/5	E	F	G	H
									O	O	O	O
23	A	5/8	B	.625	C	6.25	D	17/32	A	B	C	D
									O	O	O	O
24	E	66.2	F	2/3	G	9/16	H	.624	E	F	G	H
									O	O	O	O

25.	A	9/32	B	.375	C	5/16	D	.328	A	B	C	D
									O	O	O	O
26.	E	24%	F	1/4	G	1/5	H	9/20	E	F	G	H
									O	O	O	O
27.	A	2/5	B	39%	C	9/20	D	1/5%	A	B	C	D
									O	O	O	O
28.	E	62 1/2%	F	9/16	G	60%	H	5/8	E	F	G	H
									O	O	O	O
29.	A	1/8	B	16 2/3%	C	5/32	D	7/64	A	B	C	D
									O	O	O	O
30.	E	37 1/2%	F	11/32	G	23/64	H	31 2/3%	E	F	G	H
									O	O	O	O
31.	A	62 1/2%	B	6/25	C	5/8%	D	66 2/3%	A	B	C	D
									O	O	O	O
32.	E	76 2/3%	F	.067	G	1/3%	H	.0075	E	F	G	H
									O	O	O	O
33.	A	1.35	B	1.3%	C	.125	D	12 1/2%	A	B	C	D
									O	O	O	O
34.	E	16 1/2%	F.	.166	G	1.67	H	16 2/3%	E	F	G	H
									O	O	O	O
35.	A	.872	B	87 1/2%	C	8.76	D	875%	A	B	C	D
									O	O	O	O
36.	E	1/4	F	.12 1/2	G	20%	H	9%	E	F	G	H
									O	O	O	O
37.	A	66 2/3%	B	7/8	C	62 1/2%	D	3/4	A	B	C	D
									O	O	O	O

38.	E .05	F .3	G 3%	H 1/10	E	F	G	H
					0	0	0	0
39.	A 33 1/3%	B 2/5	C .3	D 37 1/2%	A	B	C	D
					0	0	0	0
40.	E 3/4	F 73%	G .7	H 17/20	E	F	G	H
					0	0	0	0

What value should be in the vacant space in each of the following groups? Place your answer in the space to the right of each group.

41. 3, 6, __, 24, 48

42. 7 1/4, __, 6 1/4, 5 3/4, 5 1/4

43. 6, 10, 14, __, 22

44. 1.3, 1.9, ___, 3.1, 3.7

45. 1/10, 1/5, 2/5, ___, 1 2/5

Place the letter representing the correct answer in the space to the right of each example.

46. 18 in = a) 1'8" b) 1.8 ft. c) 180% of a ft.
d) .5 yd.

47. 7 qt. = a) 28 pt. b) 87½% of a gal. c) 3½ gal.
d) 1.75 gal.

48. 20 lb = a) 2% of a ton b) 32 oz. c) .01 T.
d) 160 oz.

49. 10 min. = a) 10% of an hr. b) 1360 sec.
c) 1/144 of a day d) 1/12 hr.

50. 20 sq. ft. = a) 2 sq. yd. b) 2880 sq. in.
c) 6 2/3 sq. yd. d) 240 sq. in.

Table 12

Review Test on Arithmetic Fundamentals, Scores and Norms

	Scores of all Practical Math. Pupils	Scores of those with Weakness in Fundamentals	
11			Cases 170
12	1	1	
13	1	1	Range 12 - 49
14			
15	2	2	Median 29
16	4	2	
17	4	3	
18	1		
19	6	5	
20	9	7	
21	3	2	
22	5	3	
23	8	4	Deciles (All Pupils)
24	5	2 Median	
25	11	4	0 -- 11.5
26	6	4	1st -- 18.3
27	8	2	2nd -- 20.6
28	9	1	3rd -- 23.2
29	6 Median	2	4th -- 25.2
30	3	1	5th -- 27.2
31	6	2	6th -- 30.0
32	14	3	7th -- 31.6
33	10	4	8th -- 33.0
34	4	1	9th -- 36.0
35	6	3	10th -- 49.5
36	5		
37	2		
38	6	3	
39	3	1	
40	7		
41	4	1	
42	3		
43	1		
44	3		
45			
46	3		
47			
48	1		
49	1		
50			

constructed scale. It is well for him to understand the difference between his teacher's opinion and an objective test result based on comparison with many pupils in varying types of school systems and from all socio-economic groups. Table 10 on page 38 shows again, by the graphical method, which is far more understandable to the students than figures and statistics, that real improvement has been made by the group. The data of Table 3 shows an improvement on this retest of as high as fourteen points, and also in many cases, that those with the higher I.Q.s made greater strides than the ones in the lower intelligence group. The I.Q., as obtained from the cumulative record cards, is then of some prognostic value in dealing with these cases. It cannot be said that it correlates highly with ability to improve in a specific subject, mathematics, but that the possibilities of improvement are definitely greater than with the low intelligence group. In fact, the coefficient of correlation here is found to be only .04. We can never discard even one soul from our remedial work on the basis of this assumption but must always consider that willingness and desire to get ahead can surmount weakness in general intelligence.

Further testing is not necessary, for the time being, to keep up the incentive of the group. In fact, such a checkup might easily defeat its purpose if no real improvement were shown by it. Improvement will not be as obvious from here on as it was in the case of these first two tests. If the

results to be obtained have not been obvious to an individual pupil by this time, no further testing results can be sufficiently startling to make him aware of the possibilities for his own improvement.

CHAPTER 4

SUMMARY

A final test at the close of the year will give the individual pupils a definite measure of their own improvement. This is also quite necessary to prove the value of the program and indicate to the teacher the need for revision if such be the case. For this purpose, a retest by the "Inventory Test" that was used at the start of the year was carried out. This seemed more logical than attempting the development of a new equivalent test. Such a new test could only be proved valid after its use, thereby postponing for a year the cycle of "measure, treat, and measure" if there were inaccuracies.

Table 8 on page 28 shows the scores for the inventory test at the start of the year in one column and those on the retest at the close of the year directly to the right. From this graphic representation it is obvious that a very real improvement has occurred in a majority of cases. Table 13 on page 51 gives first the amount of improvement in each individual case. These changes range from 0 to 19 with a median of 9. Since the test contains but sixty examples, this would appear to validate the time and thought put into the program, at least for a very high percentage of the pupils concerned.

Also on table 13 have been recorded the I.Q.s opposite each improvement score. The pupil with the highest I.Q. (122) had a top increase in score (19), and the pupil with the lowest I.Q. (76) was among those having no increase in score. However,

Table 13

Change in Score on Inventory Retest
at the Close of the School Year

Increase in Score	Number of Cases	I.Q.s (Otis) for Each Case	Mean I.Q.
20			
19	3	87-98-122	102
8	1	116	116
7			
6	1	103	103
5	1	90	90
4	1	78	78
3	4	77-81-82-110	84
2	7	90-90-93-95-104-105-112	98
1	6	88-90-93-108-114-121	102
10	4	85-92-97-104	94
9	8	90-91-98-99-101-106-107-114	101
8	7	94-97-107-108-111-112-113	105
7	3	84-95-105	95
6	5	86-95-103-104-104	98
5	2	104-109	106
4	4	100-101-108-112	105
3	1	117	117
2			
+1			
0	5	76-82-88-94-101	88
-1			
2			
3			

Number of Cases 63

Range 0 to +19

Median +9

Table 14

Correlation Between Inventory Test and Inventory Retest

Inventory Test	Inventory Retest 22-25	26-29	30-33	34-37	38-41	42-45	46-49	50-53	54-57	58-61
15-17										
18-20					1					
21-23				1	2					
24-26	1									
27-29		1		1	1		1			
30-32			1		1	2				
33-35					1	3	4			
36-38						3	5	5	1	
39-41					2	1	7	9	1	1
42-45						1	11	15	1	

Coefficient of Correlation .62

the data in the column headed Average I.Q. shows practically no correlation between intelligence and improvement in mathematics. Statistically this correlation is .04. It is unfortunate that sincerity of purpose and willingness to learn are not measurable factors in this matter of degree of improvement. From the Table we see that some of the individuals with the lower I.Q.s improved far more than some of their coworkers who are in the upper levels of general intelligence. The only available explanation for this situation is the degree of "strength of purpose" involved. This is an intangible that cannot be fitted in any way into the field of statistics.

The problem of the thesis is to find a method to strengthen the weaknesses in mastery of the fundamental processes of arithmetic. Because of this weakness, many pupils taking general mathematics at the secondary school level have considerable difficulty in meeting with even a moderate degree of success. By means of a testing program and remedial work, very definite improvement was made in all but five of the cases. This improvement is shown in Table 3 by means of the decile or percentile ratings of the scores for the individual pupils. Table 8 on page 28 shows very graphically the decided change that took place for the group as a whole by means of the Inventory Test scores at the start of the year and the retest on the same at the close of the year. Table 13 on page 51 gives a quantitative measure of this improvement which is very

gratifying. The increase in score ranges from 0 through 19 with a median or average increase of 9. The test having but sixty questions, this shows a real value in the project.

The only item in the previous history of the students that has any predictive value on the results of the program was found to be the general intelligence. There is but small statistical correlation between the I.Q. and the quantity of improvement. However, the I.Q.s shown on Table 13, page 51 do indicate, indirectly, that more of the low improvement scores were made by those having the low I.Q.s.

A large part of the success of any such remedial program is dependent upon the motivation at its inception. The pupil who is weak usually recognizes the fact himself. It is for us to show valid rationalization of the weakness, and also how very simply he can overcome the condition. Encouragement can be given by the results of the testing program and a happier child, mathematically speaking will be found at the close of the year. The following table shows the testing program of the thesis in brief.

Table 15

Schedule of Tests

	Tests	When Given	Reason
1.	Inventory Test	First meetings of the year	Achievement
2.	Basic Skills in Arithmetic Test	Immediately after #1	Diagnostic
3.	Fundamental Processes Test	End of first quarter	Achievement and Motivation
4.	Retest of Basic Skills in Arithmetic Test	End of half year	Achievement and Progress Study
5.	Retest of Inventory Test	End of year	Summary of Year's Achievement

CHAPTER 5

SUGGESTIONS FOR FURTHER STUDY

Having completed the study of the specific problem with which we are concerned, it would be strange if some added questions had not presented themselves. Our problem concerned itself with enabling the pupils to be more capable of accomplishing the work of first year secondary school general mathematics.

While the methods of the thesis have accomplished a great deal toward achieving this aim, perhaps some other procedure could arrive at superior conclusions. If the school had sufficient numbers of pupils taking high school general mathematics, would a separate remedial class be more advantageous than keeping the weak students with the whole group? The ideal situation would be to test the results under both methods either at the same time or by using one method one year and the other the following one. Is there a social aspect to consider in remedial work? Would not a separate class be far superior from this standpoint alone? Administrative viewpoint is another matter to be reckoned with when it comes to establishing a different class. Would it be sufficiently worth while to engage another teacher full time, or perhaps, could this be a part time assignment?

Another question that arises in my mind is whether or not there may be some pupils in our college preparatory classes that need remedial work at the same level as our practical

mathematics students. If this could be ascertained and the groups combined, a separate remedial class would be possible in the smaller high schools.

The administrative problem of any remedial classes is open to considerable study. Some pupils may need only a few weeks of remedial work while others may require a much longer period. As some are assigned to regular classes the teacher load there may become too heavy and that of the remedial teacher too light to warrant his continuance. There are indeed many aspects of remedial class work versus remedial work in the regular class that could be studied to the advantage of the pupils concerned.

Many of the pupils in Practical Mathematics I may wish to elect another year of more advanced work in the subject. The matter of advice and guidance for these folks then becomes a new problem. We need as accurate information as possible for prognostic purposes. For the first year course in high school mathematics we tested on the material of arithmetic through percentage. Is there another step in the mathematical development of the child which can be measured? Or is there a separate phase of his general intelligence that must be considered. Drs. Thurstone and Thurstone of Chicago University have developed intelligence tests (Primary Mental Abilities Tests) which consider number concept and logical reasoning as two separate measurable factors. For my own part, I like to consider logical

reasoning as "growing up", or a step in mathematical development of the child. The two thoughts might well be intermingled in a future study or analysis.

Some method of objective testing needs to be devised to measure this ability as it relates to the field of mathematics. A pupil's rating on such a test should enable the teacher or counselor to make a definite stand as to whether he should go on to further study in this field or not. Advanced work will involve more and more problem solving of one type or another. Success in this will depend not only upon mastery of fundamental processes but particularly upon this logical reasoning process. There will be new concepts, it is true, yet their mastery will, because of their increasing complexity, depend on strength in this mental ability.

The first suggestion is then to consider separate remedial classes in general mathematics and ascertain the comparative value of the separate class to that remedial work done in connection with the regular groups. Secondly, the question arises of carrying remedial work over to some of our college preparatory pupils, thereby making separate remedial classes available in smaller schools. The third thought that should have consideration is that of measuring further development in mathematics for guidance in selection of advanced courses within the school and possibly, also, the choice of post-secondary school studies.

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