2016-07-01

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Pierre Perron, Jiawen Xu. 2016. "Comments on "In-sample confidence bands and out-of-sample forecast bands for time-varying parameters in observation driven models"." INTERNATIONAL JOURNAL OF FORECASTING, Volume 32, Issue 3, pp. 891 - 892 (2).

https://hdl.handle.net/2144/26268

Comments on "In-Sample Confidence Bands and Out-of-Sample Forecast Bands for Time-Varying Parameters in Observation Driven Models"

Pierre Perron[*] **Jiawen Xu**[†]
Boston University **Shanghai University of Finance and Economics**

Feburary 26, 2016

Cox (1981) classifies time varying parameter models into two categories: observation-driven and parameter-driven models. In observation-driven models, the parameter updating equation is a deterministic function that includes past parameters and observations. The parameters are perfectly predictable one-step-ahead given past information and the likelihood function is available in closed-form. Examples include the ARCH and GARCH models of Engle (1982) and Bollerslev (1986), and the GAS model of Creal et al (2013). On the other hand, in parameter-driven models the parameters evolve as a stochastic process with idiosyncratic innovations. A closed-form expression of the likelihood function is usually not available for this type of models. The parameters are estimated via more involved simulation-based filtering algorithms. Examples of models in this category are the stochastic volatility models of Tauchen and Pitts (1983) and the random level shift models used in Lu and Perron (2010) and Xu and Perron (2014, 2015).

The time-varying parameter models are popular especially due to their potential to overcome problems related to out-of-sample forecasting failure. In previous work, the focus was mainly on obtaining estimated parameters and achieving improved forecasts with smaller mean-squared forecast errors. To our knowledge, there is a lack of results related to in-sample confidence bands and out-of-sample forecast bands for models in both categories. The main contribution of Blasques et al (2016) is that they are amongst the first ones attempting to

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[*] Corresponding author: Department of Economics, Boston University, 270 Bay State Rd., Boston, MA, 02215; perron@bu.edu, Ph: 617-353-3026, Fax: 617-353-4449.

[†] Shanghai University of Finance and Economics, 777 Guoding Road, Shanghai, China, 200433 (xu.jiawen@mail.shufe.edu.cn).

provide methods to construct both in-sample confidence bands and out-of-sample forecast bands for time-varying parameters in observation-driven models. Moreover, they assess the relative performance of each method under different settings using Monte Carlo simulations.

In their model setting, the observations $\{y_t\}_{t=1}^{T}$ are given by $y_t \sim p(y_t|f_t;\theta)$. The time-varying parameter f_t follows the updating equation:

$$f_{t+1} = \phi(f_t, y_t; \theta)$$

where $\phi(.)$ is a differentiable recurrence function and θ is the static parameter. As the authors state, the in-sample estimation $\hat{f}_t$ for $t = 1, ..., T$ is a weighted function of f_1 and $y^{1:t-1}$ with the weights determined by $\hat{\theta}_T$. The in-sample confidence band constructed for $\hat{f}_t$ only deals with the parameter uncertainty caused by $\hat{\theta}_T$. As long as the asymptotic distribution of $\hat{\theta}_T$ is known, they can construct analytical bands by linearizing the updating function $\phi(.)$. Alternatively, they can calculate simulation-based bands when the updating function is highly nonlinear. When it comes to constructing the out-of-sample forecast bands, they need to consider innovation uncertainty together with parameter uncertainty. This problem can be solved by extrapolating multiple innovation paths for each draw of $\hat{\theta}_T^i \sim N(\hat{\theta}_T, T^{-1}\hat{W})$, where $\hat{W}$ is an estimate of the asymptotic covariance matrix of $\hat{\theta}_T$.

The methods proposed in their paper enlighten us on the same problem of constructing in-sample confidence bands and out-of-sample forecast bands in parameter-driven models. However, there indeed exist major differences between these two types of models. The updating equation (process) for the time-varying parameters in parameter-driven models can be written as:

$$f_{t+1} = h(f_t, \eta_t; \psi)$$

where $\eta_t \sim g_\eta(\psi)$ is the idiosyncratic innovation and ψ is the static parameter. The time-varying parameter f_t follows a recurrence process with its own innovations. Therefore, the in-sample confidence band needs to incorporate both parameter uncertainty and innovation uncertainty. The parameter estimate $\hat{\psi}$ is constructed via Monte Carlo maximum likelihood estimation. Let the estimate of the asymptotic covariance matrix of $\hat{\psi}$ be defined by $\hat{\Sigma} = -\{\partial^2 \log \hat{L}(\hat{\psi})/\partial\psi\partial\psi'\}^{-1}$, where $\hat{L}(\hat{\psi})$ is the Monte Carlo estimate of the likelihood function evaluated at $\hat{\psi}$. The estimate $\hat{\Sigma}$ can be computed numerically. The estimate $\hat{\psi}$ is subject to simulation errors, while $\hat{\Sigma}$ is a measure of the uncertainty that does not account for such simulation errors . However, as shown in Durbin and Koopman (1997) that effect of the latter is quite small and can safely be ignored. Once an estimate of the asymptotic distribution of $\hat{\psi}$ is obtained, the in-sample confidence band for $\hat{f}_{t+1}$ can be constructed using simulation methods similar to the filtering forecast band method proposed in their paper. The procedure can be described as follows:

1. Draw M parameter values $\hat{\psi}^i$ from the asymptotic distribution $\hat{\psi}^i \sim N(\hat{\psi}, T^{-1}\hat{\Sigma})$. Note that if the limit distribution is not Normal, one can simply use the one that applies.

2. Given $\hat{\psi}^i$ and for each time t, draw S sequences $\eta_t^1, ..., \eta_t^S$ from the estimated density $\eta_t^s \sim g_\eta(\hat{\psi}^i)$ for $s = 1, ..., S$ and $t = 1, ..., T$.

3. Given the observations $\eta_t^1, ..., \eta_t^S$, the filtered sequence f_1^s, $\hat{f}_2^s, ..., \hat{f}_T^s$ can be determined using the updating function

$$\hat{f}_{t+1}^s = h(\hat{f}_t^s, \eta_t^s; \hat{\psi}^i).$$

4. Repeat steps 2-3 for $i = 1, ..., M$ to obtain $M \times S$ filtered paths of $\hat{f}_t^{i,s}$.

5. Calculate the appropriate percentiles for each t over the $M \times S$ draws of $\hat{f}_t^{i,s}$ to obtain the in-sample confidence band of $\hat{f}_t$.

The procedure to construct the out-of-sample forecast band for $\hat{f}_{T+n}$ is actually the same as described above. We simply need to obtain $M \times S$ extrapolated paths of $\hat{f}_{T+n}^{i,s}$ to compute the percentiles. We beleive this extension of their method should prove useful for time-varying parameter-driven models.

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