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Teaching concepts of decimal fractions in grade three

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TEACHING CONCEPTS OF DECIMAL FRACTIONS IN GRADE THREE

Submitted by

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CHAPTER I

THE PROBLEM

1. Introduction

Research has brought about many changes in both the content and the method of teaching. Educators theorize, test, evaluate, and re-evaluate. Every good educational system is in a constant state of re-appraisal. Only by so doing can the American Public School System justify its existence. It is in this tradition that the present study is presented.

2. Statement of the Problem

The decimal fraction is being taught presently in the sixth grade. This paper will try to show a need to introduce the decimal fraction at an earlier grade and give more emphasis to its study and understanding all through the arithmetic program.

3. Justification

The need for scientists and mathematicians in today's world is an established fact. Our age of technology depends on well trained young men and women to fill the positions available in research, both in government and industry. The demand for capable, intelligent men and women in these fields is always great. Today 'crash programs' are being initiated in high schools and colleges to cater to this pressing need. A long-range view, however, should concern itself with the elementary school program. One segment of the elementary school arithmetic curriculum is the decimal fraction. This mathematical concept has all but revolutionized our use of, and computation with numbers. Great strides have been made in the Twentieth Century: witness the business machine or the 'electronic brain'; each based on a decimal concept. Tomorrow's adults must be ready to cope with, master, and control machines such as these. They must be well grounded in the meaning and use of the decimal fraction. It must be given more emphasis in today's schools.

4. Scope

This study concerns itself with (1) the place of the decimal fraction in the present curricula, (2) a comparison of the decimal fraction and the common fraction, and (3) the need for an earlier introduction to, and a more thorough teaching of the decimal fraction. Several introductory lessons in the decimal fraction are included.

5. Limitations

As far as the writer could determine, no authority on elementary arithmetic who is writing at present (with one exception[1/]) advocates any particular emphasis on decimal fractions in the early grades. Most advise that decimals be introduced at the end of Grade V if time permits, and be studied completely in Grade VI. This study merely indicates the feasibility of an earlier introduction to decimal fractions. No conclusions or recommendations can be final until actual testing of this theory takes place.

6. Definition of Terms

The 'broken number' was for centuries known as the fraction. With the advent of the decimal fraction, the older symbol for the 'broken number' became known as the common or vulgar fraction. In this study, to differentiate between the two types of fractions, the common fraction will be referred to as the fraction, and the decimal fraction as the decimal.

J. T. Johnson, "The Decimal Versus Common Fractions", <u>The Arithmetic Teacher</u>, (November, 1956), 3:201-206.

CHAPTER II

REVIEW OF RESEARCH

1. History of the Decimal

The need.-- The history of our number system dates back to early man. As the system evolved, it borrowed the best ideas of many peoples. Some notations, however, instead of becoming more simple, became more awkward and cumbersome. Inventions having little to do with arithmetic influenced the progress of number adversely. The invention of the printing press was one such. Tradesmen and mathematicians used this remarkable invention to lengthen the fraction to unmanageable proportions.[1] Both the tradesman and the mathematician found these fractions almost too cumbersome to manipulate. There must be a simpler method of notation.

Stevinus fills the need.-- The decimal, simple and inevitable as it seems today, did not evolve but was an invention of the Middle Ages. It was not until 1585 that a Flemish mathematician, Simon Stevin de Bruges, also

Burdette Buckingham, Elementary Arithmetic: Its Meaning and Practice, Ginn and Company, Boston, 1953, p. 343.

known as Stevinus, published a paper, expounding a new system of fractions called decimals.[1] This slim pamphlet, not more than seven pages in the French translation (La Disme) was widely read. This, then, was the beginning of what has been called "the most important step in the progress of modern arithmetic."[2]

Decimal notation.-- Decimal fractions had been employed for the extraction of square roots some five centuries before his time, but nobody before Stevinus established their daily use.[3] His method of notation, however, was unwieldly. He printed little circles around the exponents of the different powers of one-tenth. For instance 237.578 was printed 237⓪5①7②8③.[4]

The point separating the integers from the decimal fractions seems to have been the invention of Bartholomus Pitiscus, in whose trigonometrical tables (1612) it is found. It was accepted and used by Lord Napier in his logarithmic papers (1614, 1619)[5] and the decimal as we know it today was quickly accepted as a valuable tool by

1/Encyclopedia Brittanica, "Decimal Coinage", Volume 7,p.21, Encyclopedia Brittanica, The Werner Company, New York, 1902.

2/Ibid

3/ Moritz Cantor, ibid., Volume 22, p. 546.

4/Ibid. 5/Ibid.

scientists, mathematicians, and tradesmen throughout the world.

Decimal systems.-- So well aware was Stevinus of the importance of his innovation that he declared the universal introduction of decimal coinage, measures, and weights to be only a question of time.1/ He was more than prophetic. Coinage in the United States, which was made decimal in 1786, consists of the dime, ten cents; the dollars, ten dimes; and the ten-dollar bill, ten one-dollars; as the most common exchange. France introduced a decimal system of money, weights, and measures, shortly after the French Revolution. The Belgian and Swiss monetary systems were assimilated to that of France in 1833 and 1851. In 1865, France, Italy, Belgium, and Switzerland became parties to a treaty for the maintainence of a common system. Decimal coinage now exists in Germany, Russia, Holland, and Portugal as well.2/

Decimals today.-- Decimals have not become the panacea for all mathematical ills that Stevinus thought they would become. However, his notation has become an integral part of our system of computation. It would not

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1/Ibid.

2/Ibid., Volume 7, p.21.

be an exaggeration to say that the decimalized fraction has become the corner-stone of today's mathematics, science, and business. In Buckingham's words:1/

> "The algorist invented decimals so that men would be free of machines (abacus). They held good in the realm of fractions and also integers. Computing machines would never be without this system which conquers cumbersome fractions and subjects them to the rule of a decimal system."

2. Uses of Decimals

Per cent.-- Shortly after the invention of the decimal, Italian tradesmen began using a form of per cent to show a fraction to the hundredth place.2/ Today our language abounds with a casual use of the per cent in all phases of business. All adults are familiar with such phrases as 20 per cent off, 80 per cent woolen, profit of 10 per cent, 15 per cent richer in protein, saving of 40 per cent, rise of 25 per cent, 90 per cent correct, 250 per cent increase, interest at 3 per cent, and 2 per cent sales tax.3/

1/Ibid., Volume 7, p. 21.

2/E.L. Ritter and L. A. Shepherd, Methods of Teaching in Town and Rural Schools, The Dryden Press, New York, 1950, p. 377.

3/History of the per cent sign may be found in D. E. Smith, History of Mathematics, Volume 2, Ginn and Company, 1925, p. 97.

Statistics likewise uses per cents, and statistics operates in many areas of thought; in economics and sociology, in biology and agriculture, in psychology and education.[1/] Buckingham concludes:[2/]

> "It is clear, then, that percentage is an essential part of arithmetic of today. This is true not only because the businessman uses it constantly, but also because millions of people who are not businessmen but adopt some of the language of business, also use it, sometimes incorrectly."

Per cents are easy.-- The reason per cents are so commonly used today is twofold. First, per cents lend themselves easily to the comparison of dissimilar numbers. The businessman can compare the sales of all his departments; the teacher can compare the absences in her room to those of the whole school. Second, it is easier to think in terms of, and to say per cents since most often they are written more like whole numbers than like fractions. "....we experience greater difficulty in thinking and communicating with fractions than we do with whole numbers."[3/]

Per cents are decimals.-- As all adults should know, per cents are simply decimals to the hundredth place.

1/Buckingham, op. cit., p. 353

2/Ibid.

3/Herbert F. Spitzer, The Teaching of Arithmetic, Houghton Mifflin Company, Cambridge, 1954, p. 259.

In computing with per cents, of course, one reverts to his understanding of the meaning of decimals.

Computing with per cents usually means computing with decimals.[1/]

> "In the case of per cents the fraction is not indicated by a point. Yet when we wish to compute with per cents we normally....drop the % sign and insert a decimal point at the appropriate place to 'show hundredths'. We do not multiply by 12 per cent but by 0.12. We do not divide by 156 per cent but by 1.56 per cent."

Frequency of usage.-- Aside from per cents, decimals are found cited in newspapers, on cereal boxes, in text books, on gasoline station sign boards, and even on the labels of clothing. In short, everywhere one turns, the decimal is in evidence.

Spitzer mentions some ordinary uses of the decimal.[2/]

> "1. The amount of rainfall on a given day, or the water equivalent of a snowfall
> 2. The yield per acre of various crops
> 3. English equivalents of the most common metric measures, e.g., a kilometer is about 0.6 of a mile
> 4. Averages as reported in experiments
> 5. Normal body temperature
> 6. Flood data
> 7. Data on feeler gages
> 8. Measurements with rulers marked with tenths of inches."

1/Buckingham, op. cit., p. 352

2/Spitzer, op. cit., pp. 242-243.

Wilson lists the following everyday uses.[1]

"1. Railway timetable: distance between points
2. Weather report: precipitation, snowfall, height of tides
3. Sizes of tires
4. Lots of land for sale
5. Newspapers: Market quotations, baseball averages and standings, stock and bond quotations."

Decimals aid industry.-- Modern, industrial machinery would have been an impossibility, had not the decimal been an antecedent fact. Mass production depends on accurate guages which must be measured, in certain cases, to the millionth place. Some occupations which did not exist a century ago use the decimal as basic measurement.[2]

"That the outside world prefers decimals is evidenced by the fact that mechanics has decimalized the inch. The auto machine shop uses the 0.0001 of an inch in its measurements. ....engineering has decimalized the foot. The surveyor's tape is graded in feet and tenths not feet and inches. Aviation has decimalized the mile. We read of the jet bomber making a flight at 686.725 miles per hour not miles and common fractions of miles."

Other examples include the following:[3]

"Decimal fractions appear on such instruments as clinical thermometers, micrometers, rulers,

1/Guy Wilson, Teaching the New Arithmetic, McGraw-Hill Book Company, Inc., New York, 1951, p. 242

2/J. T. Johnson, "The Decimal Versus the Common Fraction", The Arithmetic Teacher, (November, 1956), 3:202.

3/Leo Brueckner and Foster Grossnickle, How to Make Arithmetic Meaningful, The John C. Winston Company Philadelphia, 1947, p. 349

> automobile speedometers, and surveyor's tapes, as well as maps and charts. ....The list can be expanded considerably."

Why teach decimals.-- It can be seen that adults encounter the decimal frequently and must have a clear understanding of its meaning and use.

A prominent educator 1/ has said: "Since adult usage favors decimals, teachers should be willing to emphasize decimals more than common fractions."

Another opinion 2/ states:

> "Uses of decimals later, in the theory and practical applications of percentage, seem to warrant thorough work in at least the few basic principles. Considering also the increasingly greater social usage made of decimals, and the prospects that the future will see a much wider use, it seems that our very best efforts should be put into its teaching."

It follows, therefore, that more time should be accorded to training children in the understanding and use of decimals than is presently given them by most of today's curriculums.

3. Grade Placement in Arithmetic

Criteria.-- It is axiomatic to say that some criteria must be established by administrators or chaos would result in the nation's schools. The three criteria presented by

1/Johnson, op. cit., p. 203   2/Ritter, op. cit.,p.376.

Morton[1] in relation to arithmetic subjects seem a reasonable and desirable one to follow. The three criteria are:

1. The logical criterion, which has reference to the structure and organization of arithmetic as a science
2. The social criterion, which indicates a concern with the usefullness of arithmetic in life's affairs
3. The psychological criterion, which is concerned with how children learn.

The logical criterion can be described as that which follows the form of a pyramid, building concepts upon that which has already been learned. Thus:[2]

> "An understanding of the decimal nature of the number system must precede attempts to understand carrying in addition, borrowing in subtraction, the placing of partial products in multiplication, and other phases of the fundamental operations with whole numbers."

The social criterion[3] demands that "the needs of learners should be satisfied as soon as it is feasible to do so."

1/Robert L. Morton, "The Place of Arithmetic in Various Types of Curriculums", Arithmetic, 1949, Monograph Number 70, University of Chicago Press, Chicago, 1949, p. 1.

2/Ibid., p. 3

3/Ibid., p. 5.

The third criterion, the psychological, is explained by Morton[1/] in these words:

> "I have suggested that we should base new learning on related old learning; build understandings gradually, one step at a time; lead pupils to discover new truths for themselves; avoid mechanical tricks and devices; provide for telescoped reteaching; and take a realistic and intelligent attitude toward teaching."

Using criteria.-- At one time the logical criterion was the only one considered in building the curriculum. This system, usually termed 'traditional', did not take into account the pupils' needs and interests. Morton[2/] notes that arithmetic does not always "....fall into a rigorously logical pattern. This is a fortunate condition, for it permits certain adjustments to suit the learners' needs and interests."

All three criteria need not coalesce at one point to indicate the exact moment a particular topic must be taught. If there is a seeming conflict, then the curriculum planner must choose that criterion which seems most important.

Sina Mott[3/] poses this problem:

1/Ibid., p. 18.

2/Ibid., p. 21.

3/Sina Mott, "Letting Arithmetic Function in the Primary Grades", Ibid., p. 24.

"Suppose it is a cold day, and the nursery-school children decide it would be nice to have warm cocoa instead of cold milk. Should the teacher say: 'No, my dears, we don't study liquid or dry measure until the fourth grade. We will wait six years, my dears."

The fact is, then, that conflict exists among the criteria. Morton 1/ says that those who are charged with the responsibility of building a curriculum in arithmetic are wont to complain that serious conflicts arise among these criteria. Sometimes practices must be determined in the light of one criterion to the exclusion of the others. An examination of curriculums in common use lends support to this contention.

When to teach.-- An attempt has been made by educators to find the optimum age at which to teach arithmetic concepts. A 'Committee of Seven' 2/ worked for a period of years in hundreds of cities trying to determine the best mental age at which a pupil can learn various topics successfully. (For example, they found the mental age for fractions to be 10 to 11 years; unlike fractions, 14 to 15 years, and so on.) As a result of this study, the so-called

1/Morton, ibid., p. 1.

2/C.W. Washburn, National Society for the Study of Education, "The Grade Placement of Arithmetic Topics: A 'Committee of Seven' Investigation", Report of the Society's Committee on Arithmetic, Twenty-ninth Yearbook, 1930, Part I, Public School Publishing Company, Illinois, pp. 641-670 'stretched out' curriculum came into being. Courses, previously taught at an early age, were now being deferred for several years.

However, there were some educators who disagreed with these findings. Brownell[1/] analyzed the weaknesses of the Report and found the mental age standards to be of doubtful value. He felt that meaningful teaching of subject matter at lower grade levels would have lowered the grade levels and mental age levels considerably.

What to teach.-- Most educators emphasize the importance of spending more time in developing meanings at each stage of the learning process.

Thiele[2/] says that "arithmetical learning represents a building-up process. Children in the middle grades are often overwhelmed with the complexity of common and decimal fractions and per cents."

Which is more important, the idea upon which arithmetic is based, or a facility of computation? Both, of course. But more important is the idea, because once the

1/William A. Brownell, "A Critique of the Committee of Seven's Investigations on the Grade Placement of Arithmetic Topics", Elementary School Journal (March, 1938), 38:495-508.

2/C. L. Thiele, "Teaching Common and Decimal Fractions and Per Cent: General Issues", Arithmetic 1947, Monograph Number 63, University of Chicago Press, Chicago, 1947, p. 32.

theory is understood, facility will follow naturally.

It is possible that we deal too much and too soon with figures and not enough with ideas. Thiele[1] thinks this is so. He contends that the issues of arithmetic are related to a greater or less degree to ideas. These ideas represent the finest thinking of the human race and as such, assume importance in the teaching of arithmetic to boys and girls.

Glennon and Hunnicutt[2] agree that the teaching of meanings comes first and remains foremost in the teaching of any arithmetic topic. They give these reasons:

> "1. Arithmetic can function in intelligent living only when it is understood.
> 2. Meanings facilitate learning.
> 3. Meanings increase chances of transfer [of learning].
> 4. Meaningful arithmetic is better retained and is more easily rehabilitated than is mechanically learned arithmetic.

<u>Why teach meanings</u>.-- Glennon and Hunnicutt[3] admit that it takes more time at the beginning of a new process to teach meanings. But they feel that whatever extra time is required at the outset to teach meanings is more than

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1/Ibid.

2/Vincent J. Glennon and C. W. Hunnicutt, <u>What Does Research Say about Arithmetic?</u>, Association for Supervision and Curriculum Development, National Education Association, Washington, D. C. 1952, pp. 14-15.

3/Ibid.

regained later on, through quicker and more intelligent learning. The effects of learnings are cumulative: their contributions to learning increase in amount as they enable the learner to gain new insights, to discover short cuts, and to apply in new ways what one has learned.

4. Grade Placement of Decimals

'Stretched out' curriculum.-- Some evidence exists that stretching out the curriculum brings its own ills. While it is certainly true that[1/] "efforts to crowd learning into a short period of time have been disastrous for all but the more able pupils", it is also true that before children have been taught certain concepts in school, examples of these concepts have appeared in their reading and their daily lives.

An example of the time lapse between need and formal teaching is found in Spitzer[2/]. Per cent in most schools is introduced in seventh grade, but science and social studies texts use per cent frequently in their fifth and sixth grade texts. The meaning of per cents must, then, be discussed in those grades regardless of where they are placed in the arithmetic curriculum.

1/Morton, op. cit., p. 264.

2/Spitzer, op. cit., pp. 257-261. adapted.

Intermediate grades suffer.-- The hours allotted to arithmetic in the intermediate grades allow the teacher little time to develop or interpret meaning. "The total[1/] time available for instruction in arithmetic in the middle grades is in the neighborhood of 100 to 125 hours per year."

The teaching-load in the middle grades is burdensome, indeed, if the following remarks, taken from Arithmetic 1949,[2/] are considered carefully.

> "In one sense, however, the work of the primary grades is preparatory for the middle grades. No ordinary child is adequately prepared to deal with the quantitative aspects of daily living at the close of Grade III. He has a small store of number concepts and there are a few simple things he can do."

Common fractions versus decimal fractions.-- Despite adult usage of the decimal, despite a crowded middle grade curriculum, despite the findings of research[3/] "in which the topics are thought of as extending over several grades, and taught meaningfully....", educators still advocate that all the work in decimals be accomplished in Grade VI. In the case of decimals, practice has not followed theory. All seem to agree that they are more convenient to write, easier to interpret, and cause less error in computation

1/Maurice Hartung, Arithmetic 1949, op. cit., p. 81.

2/Ibid., p. 80.

3/Glennon and Hunnicutt, op. cit., p. 18.

than fractions. Yet, "civilization holds to the clumsy common fractions and spends almost endless time and limitless energy in trying to teach and use them."[1/]

Morton[2/] proposes that decimals representing tenths and hundredths be introduced late in the program of the fifth grade. "The treatment of decimals in this grade should be brief and simple, but may be ommitted if there is insufficient time."

Morton's[3/] treatment of fractions is not so cursory. He notes that once fractions were taught in Grade V, but today the fraction is introduced in Grade I. Experiences with fractions continue in Grades II, III, and IV because "fractions occur in normal experiences of children whose ages range from six to ten".[4/] Morton[5/] concludes that it is possible for children of these earlier ages to understand fractional concepts if the learning is carefully planned and skillfully executed. The child, having had four years of background in fractions, is taught computation of fractions in Grades V and VI.

Although the above remarks refer to fractions, they could easily have applied to the teaching of decimals.

1/Ritter and Shepherd, op. cit., p. 368.

2/Robert Lee Morton, Teaching Arithmetic in the Elementary School, Silver Burdett Company, New York, 1953, p. 320.

3/Ibid., p. 263. 4/Ibid. 5/Ibid.

In the following quotation, Spitzer[1] also refers to fractions. The writer has inserted the word 'decimal' to emphasize the similarity of concepts which should apply in teaching both of these topics.

> "It is the purpose of the work with fractions [decimals] in the primary grades to provide children with knowledge and ideas which will be of use at the time and which will provide a good background for later learning of computation procedures with fractions [decimals]."

Experts differ.-- Whereas Morton would begin the teaching of fractions in Grade I and continue until computation is taught in Grade V, he would limit the teaching of decimals to one grade, Grade VI. Wilson[2] would omit all drill on manipulation of decimals. He says that United States money and percentages can be covered incidentally by using functional problem units. He[3] finds that "decimals, aside from United States money, call for reading knowledge only. Beyond this, a small amount of informational attention should meet all requirements."

Wilson's statement seems to be in direct conflict with that of Thiele,[4] who says: "Among adults, few indeed

1/Op. cit., p. 205.
2/Op. cit., p. 256.
3/Ibid. p. 91
4/Op. cit., p. 23.

are those who have completely mastered common and decimal fractions and per cent."

The above statement could easily be taken as an indictment of today's schools and their methods of teaching. Why is such a basic concept of the decimal number system not understood? Perhaps it is because the educator has isolated the decimal; he has tried to teach abstract theory, practical meaning, application, and mahipulation of the decimal, all in the space of one year or less. It has been said[1/] that when "any section of mathematics such as decimal fractions is chopped off and interpreted in isolation from the total system of concepts, there is little chance for intelligent understanding of the topic."

5. Decimals Should Precede Fractions

Logical criterion.-- According to the logical criterion, the proper place of decimals in the teaching scheme would seem to come directly after the teaching of whole number concepts. Place value is of prime importance in both integers and decimals. Johnson,[2/] a firm believer that decimals should take precedence over fractions in our teaching system, says:

1/E. Spencer and M. Brydegard, Building Mathematical Concepts, Henry Holt and Company, Inc., New York, 1952, p. 302.

2/Op. cit., p. 202.

> "We have to have our children learn all of the four operations in addition, subtraction, multiplication, and division not only in whole numbers where place value is the central theme of understanding but in common fractions where there is no place value [thus] causing the operations to be performed in an entirely different manner, subject to rules altogether different, when there is a simpler way that operates by the principle of place value, thus using and re-enforcing a technique which is already known."

<u>The psychological criterion</u>.-- H. G. Wheat,[1/] in discussing the teaching of decimals, says: "The pupil can proceed to learn something that at the outset appears new, but that is really only a new way of using ideas he has been gaining in his preceding work."

According to the psychological criterion, then, the teaching of decimals, at least decimal concepts, should follow the teaching of integer concepts.

<u>Social criterion</u>.-- Children have a need and a use for decimals long before formal teaching of them begins. The decimal point is encountered as soon as the dollar sign is used in writing amounts of money. As Spitzer[2/] says, "The foundation experiences with decimals, like other phases of arithmetic in a good program, begin long before the pupil starts systematic study of decimals."

1/H. G. Wheat, <u>The Psychology and Teaching of Arithmetic</u>, D. C. Heath and Company, Boston, 1937, p. 403.

2/Op. cit., p. 242.

An earlier teaching of the meaning of decimals seems indicated. This is not to say that all teaching of the fraction should be deferred until after decimals have been completely taught. The theory of the 'broken number', that which is less than whole (or less than an integer), is basic to the understanding of both topics. The need of the primary school child to understand the idea inherent in the symbol ½ extends to his need to understand the meaning of the decimal point.

Pupils prefer decimals.-- The following data is taken from Johnson's report in the Arithmetic Teacher of an experiment he conducted at the Harvard School in Chicago. The experiment was made to determine which were the easier to learn and use, the decimal or the fraction. A fifth grade 'B' class was taught the following most commonly used fractions[2] and their decimal equivalents.

1/2 = 0.50	1/5 = 0.20	1/3 = 0.333
1/4 = 0.25	2/5 = 0.40	2/3 = 0.666
3/4 = 0.75	3/5 = 0.60	1/6 = 0.167
	4/5 = 0.80	5/6 = 0.833

1/Op. cit., pp. 201-208.

2/"The fraction one-half alone makes up 60 per cent of adult fractional usage. Halves, fourths, and thirds make up 90 per cent." Wilson, op. cit., p. 91.

After fourteen weeks of learning these combinations and using them in short addition and subtraction examples, a test in fractional computation was given the '5B' class and the '5A' class as a control group. The experimental group changed the fractions first then added or subtracted. The control group found common denominators. Here is a sample of each kind of example.

Add:	Subtract:
3 1/2	4 3/4
<u>4 1/4</u>	<u>1 1/8</u>

Although the mental age averaged about 0.4 years in favor of '5A', the results of the test favored '5B'. Here are the results:

1. Accuracy: 19 per cent in favor of '5B'.
2. Time: 12.5 minutes in favor of '5B'.

For purposes of accuracy, the experiment was performed again, this time with a control group three semesters above the grade level of the experimental group. The best '6A' class in the school was chosen.

Here are the results:

1. Accuracy: 7.6 per cent in favor of '5B'.
2. Time: 3.5 minutes in favor of '5B'.
3. Mental Age: 1.4 years in favor of '6A'.

Decimals easier.-- The results of this test are not conclusive, of course. But they do indicate an ease of computation with decimals which do not obtain when equivalent fractions are used. Consider the example

```
-245 1/5
 167 3/4,
```

which has to be changed to

```
245 4/20
167 15/20
```

and then to

```
244 24/20
167 15/20.
```

Compare it to

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245.2
167.75,
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in which the process is hardly more complicated than subtracting with whole numbers.

Johnson 1/ sums up his reasons for an earlier teaching of decimals thus:

> "1. After United States money is taught it is an easy step to one- and two-place decimals, instead of breaking in with common fractions with entirely new operations. There is no place value in common fractions, hence an entirely new system of procedure has to be taught throughout..../the four processes/ before any of these four fundamental operations are mastered.
>
> 2. Needed continuation and re-enforcement of place value concepts takes place after operations in whole numbers have been only partly learned.
>
> 3. It is far easier to find and correct errors for both teacher and pupil.

1/Op. cit., p. 203.

6. Conclusions

Research indicates that decimals are important to industry and science. They must be used and understood as a part of the daily life of adults. Even our monetary system is based on the decimal. According to the criteria set down for the teaching of arithmetic subjects, the teaching of the decimal should follow the teaching of the four processes with whole numbers; because

1. it avoids breaking in with a new way of computing,
2. it cements that which has already been learned about the decimal nature of the number system,
3. it makes use of previous knowledge by using the same addition, subtraction, multiplication, and division facts,
4. it fills a definite need in the child's life, and
5. it teaches that which will be of continued use to the child as he grows up.

Decimals are taught presently in Grade VI. Teachers in the middle grades have a great amount of factual matter to teach. Their time is limited. Teaching the meaning of decimals in the primary grades would satisfy the theory of the 'stretched out' curriculum, provide carefully planned, meaningful teaching of decimals so that the child will already know basic concepts before he reaches the intermediate level.

Decimals are easier to work with, and not as cumbersome as fractions. They allow for more simple computation procedures, and permit an easier check-back for errors.

All of these findings indicate to the writer a definite need for more emphasis in teaching the decimal, more time given over to its teaching, and an earlier introduction to the meaning of decimals to school children.

CHAPTER III

THE TEACHING SUGGESTIONS

1. Requisite Learning

Positional value.-- In order to understand decimals, children must have a clear and definite knowledge of the positional value of integers and an understanding of fractional concepts. These ideas can be reviewed and re-examined in the first half of the third grade. Spencer[1] states the concepts which can be stressed.

> "The positional direction for expressing decimal integer values is to the left. Each number space to the left of the units' place represents a value that is ten times as large as that represented by the space to its right. Conversely, each number space to its right represents a value ten times as small as the value represented by the space to the left. These comparative expressions, "ten times as large as" and "ten times as small as" are extremely useful in interpreting decimal values."

As a background to the teaching of decimals, these basic arithmetical concepts must be understood.[2]

1. There are ten primary digits in the decimal system of notation.
2. Number has a positional value. In the expression "33", one is aware that the "3" at the left has a

1/Spencer, op. cit., p. 158. 2/Ibid., p. 186.

number value of 30, or three tens, and that the "3" at the right represents a number value of three units.

3. A given digit in the tens position has a value ten times as large as if that digit were in the units' position.
4. A given digit in the units' position has a value ten times as small as if that digit were in the tens' position.

2. Three Points to Teach

Extention of number system.-- Prior to computing with decimals, the most important feature to impress upon young children is "that decimal fractions are simply an extention of our decimal number system to the right of the decimal point."[1]

Base position.-- The second important point for children to learn is that the units comprise the base position. Spencer[2] discusses the importance of this concept.

> "The units' position in a decimal number is the base position. In that position is expressed the primary value of each of the digits. All the other positions in the number symbol drive their values through positional cues from the units' place. In terms of positional value, the units' place is the "point of origin."

1/Morton, Teaching Arithmetic in the Elementary School, op. cit., p. 321.

2/Spencer, op. cit., p. 158.

Decimal point.-- The third point, aptly enough, is the decimal point. It is most important that students understand the importance and the function of the decimal point. Spencer[1/] defines the decimal point thus: "The decimal point is a device to indicate the location of the units' position. The decimal point is as truly a part of the decimal system of symbols as are 0,1,2,3,4,5,6,7,8,9."

Units, then, are the base position; and the units' place is at the extreme right. If, however, the units are at any other place than the e xtreme right, then units are shown through the use of a point directly to the right of the units number. The decimal point serves a locative function only. It locates the units' position.

The three points to be taught primary children, are, then:

1. Decimals are an extention of integers.
2. The units position is the base position.
3. The decimal point locates the units position.

3. Vocabulary

New vocabulary is built slowly, and s ince there are relatively few new terms involved in teaching decimals, they can be learned concommitantly with the new learning.

1/Ibid., p. 159.

These terms should, however, be tied in with past learning. Wheat[1] introduces the study of decimals in this way.

> "In your study of fractions, you have learned about parts of many different sizes, such as halves, thirds, fourths, fifths, tenths, twelfths, etc. You found it difficult sometimes to compare them because the parts were of different sizes, and because the parts to be compared were often not related. Now you are about to study about parts that are of only three different sizes- tenths, hundredths, thousandths- and, what is more, about parts that are closely related."

Wheat[2] goes on to explain that the fractions to be studied are called decimals, that the word decimal may be new but the children really began studying the meaning of the word as far back as the first grade. The word decimal was taken from a word, decem, meaning ten. The month of December used to be the tenth month. A decade is ten years.

Morton[3] names the following words as necessary to the study of decimals: decimal point, common fraction, and decimal fraction.

4. Procedure

A posteriori learning.-- Children learn about decimals in exactly the same way they learn other arithmetical concepts, that is to say, through an orderly series of experiences which begin with concrete objects and progress toward abstractions.

1/ Wheat, op. cit., p. 403 2/ Ibid.

3/ Op. cit., p. 320.

An orderly plan.-- Grossnickle[1] identifies five developmental stages in a planned series of experiences leading to learning.

1. Readiness for learning
2. Laboratory period for discovery
3. Verbal and symbolic representation of a quantitative situation
4. Systematic verbal presentation, and
5. Adult level of operation.

Primary grade children can be introduced to the first two points of this series.

Instructional materials.-- In any learning situation the teacher is aided by choosing the proper instructional materials. Grossnickle[2] classes instructional materials into four parts; (1) real experiences, (2) manipulative materials, (3) pictorial materials, and (4) symbolic materials.

How can real experiences be used to reach the meaning of decimals? The starting point would be the experiences of the children, themselves, including their experiences

1/Foster E. Grossnickle, et. al., "Instructional Materials for Teaching Arithmetic", The Teaching of Arithmetic, Fiftieth Yearbook, Part II, National Society for the Study of Education, University of Chicago Press, Chicago, 1951. pp. 155-185.

2/ Ibid.

with money. Thus, "artificial devices employing counters, or the like, cannot be considered as practical as pennies, dimes, dollar bills, which are familiar to children."[1]

Morton[2] also advocates the use of money to teach decimals, but would begin the teaching with a demonstration of the automobile adometer.

Real and manipulative materials.-- United States money and the odometer both constitute real and manipulative materials. Other materials of this nature were used by Frank Arnold,[3] principal of Public School 148, New York City, in a demonstration lesson in decimals. As a preparation for the lesson, Arnold's students visited a gasoline station. They examined calcometers; observed the way calcometers record the amount of gasoline and its cost. The children observed the odometer of the automobile. When they returned to school, they made scale models of odometers that "worked". The teacher provided a real odometer for the children to manipulate.

The odometer is useful because it shows tenths and provides abundant uses for this largest decimal.

1/ Thiele, op. cit., p. 32.

2/Op. cit., p. 320

3/ Frank Arnold, "The Decimal Is More than a Dot", The Arithmetic Teacher (October, 1955), 2:80.

Pictorial Materials.-- Since the tenths figures on mileage meters are often a different color from other figures, this special way of showing a fraction of a mile should be used in bulletin board and chalk board statements of distances measured.

Another pictorial representation of use in teaching decimals is a diagram[1/] showing one hundred squares, ten across, and ten, down. This diagram can be made on the chalk board, mimeographed, or separate squares can be cut out and placed on the felt board.

If mimeographed, it can be used in the following way. Children will be instructed to color in fifty of the squares, constituting one-half of the diagram, or 0.50. At another time, they will be instructed to color in twenty-five of the squares, or one-fourth, or 0.25. This exercise can be continues with other parts until the children are aware of the relationship between the parts and the whole. It also shows the relationship between fractions and decimals.

The decimal concept will be more meaningful, if it is related to the children's understanding of fractions. That is, children who have previously connected the

1/ Irene Sauble, "Teaching, Fractions, Decimals, and Per Cents: Practical Applications", Arithmetic 1947, op. cit., p. 42.

symbol $\frac{1}{2}$ with two equal parts of a whole must now enlarge that concept to include a base of ten or one-hundred. Thus, half the pegs on a standard peg-board is fifty and the teacher should not hesitate to write it 50/100 and compare it side by side with 0.50. A constant comparison of the fraction and decimal symbols should be made if the decimal concept is to have a real meaning.

Abstract thinking.-- As a direct preparation for decimals, children must be taught to identify and name the place held by a number. They must also know the ratios involved in place value. Using the number symbol 111, the place of each number should be identified first as one hundred, one ten, and one unit.

Using the same number, the ratios are discussed in this way. The teacher should try to elicit from her pupils the fact that the '1' in the tens place has ten times the value of the '1' in the ones place. The '1' in the hundreds place has ten times the value of the '1' in the tens place. Conversely, the '1' in the ones place is ten times smaller than the '1' in the tens place. If these relationships between the place numbers cannot be elicited from the group, then the teacher must initiate the process. One way to do this is to place a single number, such as a three, on the chalk board. The teacher will ask the students to add a number to the three which will make it ten times as large. This short exercise can be used as many times as necessary to instill the idea of ratio.

Exercises.-- The following exercise was taken from an article by Irene Sauble.1/

Three boxes of different sizes are placed on a table. The small box contains packages of single sheets of paper. The next larger box contains packages of ten sheets each. The largest box contains packages of ten packs of ten sheets each.

1. Take one package from the hundreds box.
   This package contains one hundred sheets.
2. Take one package from the tens box.
   This package contains ten sheets.
3. Take one sheet from the ones box.

In all, one hundred, one ten, and one sheet are removed. This contitutes 111 sheets.

This 111 means 100.

This 111 means 10. Ten is 1/10 of 100.

This 111 means 1. One is 1/10 of 10.

Going from left to right in the number 111, each place has a value of 1/10 the value of the place next to it.

1. Take a single sheet out of the ones box and cut

1/ Ibid., p. 43

it into ten equal parts. Each part is 1/10 (0.1). Add another, smaller box to hold tenths.

2. The boxes at the right of the ones box contain parts of the one whole. Take 1/10 from the tenths box to put with the original number. Now the number is 111 1/10. Write it 111.1. The decimal point means we have come to the end of the whole numbers. Everything written at the right of the decimal point stands for parts of the one whole.
3. Several tenths are taken out of the box, added together, written by the teacher on the chalkboard, and the symbol read by the children.

Games.-- The following game was taken from an article by Richard Ernest.[1]

On cards print the numbers from one to nine, several zeros, and a decimal point. The cards should be made from heavy white poster paper.

Procedure: Divide class into two sections; one section at the front of the room holding the cards. Before starting the game, each child announces what he is worth. The 'decimal point' should say he has no value; the zeros are place holders.

1/ Richard Ernest, "Introducing Mr. '0' and Mr. Decimal Point", The Arithmetic Teacher (November, 1956), 3:206.

The teacher begins by calling simple numbers which the children form by taking the proper formation. Presently, the teacher calls out a decimal fraction.

At this time the game stops while the children decide what the decimal point does to a number, and whether it has any intrinsic value. The teacher can liken the decimal point to a signpost. It separates the whole number from the fraction. Pupils can readily see that the point has no value and does not hold a place.

In this game, the pupils can intellectualize and verbalize the ideas relative to the decimal point. This game also gives practice in reading decimals.

SUGGESTIONS FOR FURTHER RESEARCH

1. Try out the exercises and teaching procedures in a third grade classroom.
2. Try out the exercises and teaching procedures on third grade children of above average intelligence.
3. Do further investigation into authoritative sources.

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