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ENACTED TASK DESIGN: TASKS AS WRITTEN IN THE CLASSROOM

Leslie Dietiker — Boston University
Aaron Brakoniecki — Boston University
Elyssa R. Miller — Boston University
Andrew S. Richman — Boston University

This paper presents and describes the construct of enacted task design, which considers the way tasks are "written" (designed) by teachers. Two enactments by different teachers based on the same written algebra task were analyzed and compared using the math story framework (Dietiker, 2015). Variations in these stories highlight four dimensions of the teacher's design work.

The tasks used in a lesson are instrumental for student learning and are transformed as they are enacted by teachers and students in the classroom (Heck et al., 2012). The teacher's role in this transformation has been framed in a variety of ways, from "offloading" to "improvising" (Brown, 2009). Although "task design" is often used to refer to the creation of static curricular forms (e.g., written textbooks), we instead focus on the ways in which a written task is further "designed" by the teachers in the classroom. That is, we interpret teachers not as curriculum users or implementers, but as purposeful designers (Remillard 2005, Brown 2009).

Accordingly, we introduce and discuss the construct of *enacted task design*, which we define as the intentional shaping of mathematical tasks by teachers and students during the process of enactment. We do this by drawing from literary theory with the metaphor of mathematical curriculum as story, highlighting differences within the manner in which the content emerges throughout enacted lessons based on the same written task. Contrasting the enactment of tasks based on identical, written curriculum materials provides a unique opportunity to identify the design work of teachers.

THEORETICAL FRAMEWORK

In order to recognize enacted lessons as mathematical designs, this paper frames each lesson as a *mathematical story* (Dietiker, 2015). Not limited to story problems, this framing foregrounds how mathematical content unfolds over sequential portions of the lesson ("*acts*"). In this sense, mathematical objects (e.g., a point) in the lesson are construed as *mathematical characters*, while processes that act on these characters are interpreted as *mathematical action*. The mathematical representation where the characters and action are found, such as on a graph on graph paper, is recognized as the *mathematical setting*. The takeaway message of a mathematical story is its *moral.*

This framework, influenced by Bal (2009), enables the analysis of how a mathematical story can generate suspense and surprise. The notion of mathematics curriculum as a story allows for not only the focus on the logical (i.e., what is known to be true), but also the aesthetic (i.e., what is felt). Thus, we conceptualize the potential tension felt by students between what is known and not known as the *mathematical plot*. To describe this tension, Barthes' (1974) offers analytic codes: question formulation, promise of an answer, snare (misleading direction), equivocation (misleading ambiguity), jamming (the question is unanswerable), suspended answer (the delay of the answer), partial answer (progress), and disclosure of the answer. The transition from the formulation of a question to its answer (if an answer is ever reached) is referred to as a *story arc*.

CONTEXT OF THE ANALYSIS

This paper presents part of a contrasting case study of two enactments of a task involving the number of points needed to determine a parabola (reprinted in Figure 1). This task, part of a lesson introducing the Zero Product Property, was selected because there were evident differences in how each task as designed was experienced by students.

How many points do you need in order to sketch a parabola? 1? 10? 50? Think about this as you answer the questions below. (Note: A sketch does not need to be exact. The parabola merely needs to be reasonably placed with important points clearly labeled.)

a. Can you sketch a parabola if you only know where its y-intercept is? For example, if the y-intercept of a parabola is at $(0, -15)$, can you sketch its graph? Why or why not?

b. What about the two x-intercepts of the parabola? If you only know where the x-intercepts are, can you draw the parabola? For example, if the x-intercepts are at $(-3, 0)$ and $(5, 0)$, can you predict the path of the parabola?

c. Can you sketch a parabola with only its x-intercepts and y-intercept? To test this idea, sketch the graph of a parabola $y = x^2 - 2x - 15$ with x-intercepts $(-3, 0)$ and $(5, 0)$ and y-intercept $(0, -15)$.

Figure 1. Parabola sketch task from Core Connections Algebra (CPM, 2012, p. 391)

These enactments were observed and videotaped in spring, 2015. The selected teachers, Mr. J and Mrs. W, were from different parts the United States and had 8 and 20 years of teaching experience, four or more of which were with the Core Connections Algebra curriculum materials. The transcribed videos were coded for their mathematical plot using Barthes' (1974) codes. Pairs of researchers coded separately and then met to resolve differences.

ENACTED MATHEMATICAL STORIES

To start, we describe the mathematical stories enacted in Mr. J's and Mrs. W's classrooms. We then present our coded diagrams that represent the mathematical plots (as we interpreted the lessons) and offer a brief explanation of how to read the diagrams.

Mr. J. Mr. J asked his students to "sketch a parabola that has a y-intercept at (0,-7) and a vertex somewhere else" and then compare their parabola with a partner to decide if they are the same parabola or not. Next, he prompted students to sketch a parabola through the points (-1,0) and (5,0) and again compare with a partner. He then asked students to sketch a parabola through all three points and compare with a partner. After this third sketch, most students indicated that they were the same. Mr. J confirmed that this should be the case, and then asked the class, "how many points are needed to draw a parabola?" The students responded "three" and Mr. J confirmed that they were correct.

Mrs. W. Mrs. W started by announcing that she was thinking of a parabola passing through (0,2) and asked students to "sketch the parabola I'm thinking of." Some students asked questions such as "Wait, that's the only clue we get?" or said "it could be so many things!" While circulating, Mrs. W indicated "nobody got it." Next, Mrs. W said her second parabola passed through (-3,0) and (-1,0) and prompted students to guess this parabola. This time, two students correctly guessed, to which she responded "nice job" and the students celebrated "Woo!" Mrs. W said her third parabola passed through (-3,0), (-5,0) and (0,-15). This time most students successfully sketched her parabola. She then revealed her parabolas and said "So the first example when I gave you the y-intercept, you all kind of had the same reaction of 'was that it?' cause I didn't give you enough information. The second one ... was that enough information?” Students replied “no” to which Mrs. W replied “It really wasn't, basically, these ladies got lucky... Cause I didn't really give you enough information. But, in the third one, where I gave you two intercepts, two x-intercepts and the y-intercept, a couple of you were able to come up with the exact graph. And that wasn't just by chance.”

Representation of Plots. Figures 2 and 3 show the plot diagrams that emerged from our coding of these lessons as mathematical stories. Each column is an act in the story. Each row contains a question that was raised and considered in the story. Shading in an act indicates that a question is still open and under consideration in a particular act. The specific codes describing the nature of the progress on the questions are represented with numbers: 1=question raised (teacher or text), 2=question raised (student), 3=partial answer (student), 4=partial answer (teacher), 5=promise of answer, 6=ambiguity, 7=jamming, A=disclosure (teacher), B=disclosure (student).

	ACT	1	2	3	4	5	6	7	8	9	10	11	12	13
	Time	01:20	04:56	07:00	11:07	15:10	16:22	17:20	20:20	23:05	25:22	27:08	33:00	34:47
	Question\Elapsed Time	03:36	02:04	04:07	04:03	01:12	00:58	03:00	02:45	02:17	01:46	05:52	01:47	05:51
1	What is the zero product property?	1C									3		A	
2	What are roots?	1											A	
3	How can I find quadratic roots?	1						43		34	3	34	A	
4	How do you sketch a parabola with the y-intercept (0,-7) and a vertex elsewhere?	1B												
5	What does a parabola look like?	1	A											
6	How exact does a sketch need to be?	1	34	4										
7	Do all parabolas drawn with the same y-intercept look the same?	13												
8	How do I sketch a parabola with x-intercepts at (-1,0) and (5,0)?		1B											
9	How could we make the parabola look better?		13A											
10	How does finding the "middle" help sketch a parabola?		13A											
11	Does a parabola stop at the x-intercepts?		13											
12	Do all parabolas drawn with the same x-intercepts look the same?		143A											
13	How do I sketch a parabola when given 2 x-intercepts and the y-intercept?			13										3A
14	Do all parabolas drawn with the same x-intercepts and y-intercept look the same?			13A										
15	How many points are needed to draw a parabola?			13A										
16	Where is the line of symmetry on the graph?			13A										
17	What is the equation of the line of symmetry?			13A										
18	How could we get the y-value of the vertex of this parabola without an equation?			15										

Figure 2. Coding of Mr. J’s Questions during the first three tasks.

	ACT	1	2	3	4	5	6	7	8	9	10	11	12	13
	Time	01:36	05:37	08:26	12:09	13:48	15:36	21:05	23:39	25:33	27:55	28:41	35:22	38:05
	Question\Elapsed Time	04:01	02:49	03:43	01:39	01:48	05:29	02:34	01:54	02:22	00:46	06:41	02:43	00:28
1	What is the zero product property?	15C								3	4	2A		
2	What characteristics does my sketch need to count as a correct match?	143	3	34										
3	What information is sufficient to sketch a mystery parabola?	163	63	4		3A								
4	Does a sketch need to have axes?	2A												
5	What should my coordinate axes look like?	24												
6	How can a student sketch a specific parabola when only given the y-intercept?	16374				3A								
7	What parabola is the teacher thinking of (with y-int at 2)?	1374				2A								
8	Is the y-intercept enough information to sketch the teacher's parabola?	263				3A								
9	Was the teacher's parabola sideways?	2A												
10	How can a sketch a specific parabola when only given the two x-intercepts?		13	34	3	3A								
11	What parabola is the teacher thinking of if the x-ints are (-3,0) and (-1,0)?		13A											
12	Are the x-intercepts enough information to sketch the teacher's parabola?		163			3A								
13	What does a parabola need to have at its ends?		13A											
14	How can a student sketch a specific parabola when only given the two x-ints and th			134	3A									
15	What parabola is the teacher thinking of if the x-ints are (-3,0) and (5,0) and y-int is			134	A									
16	Are the x-intercepts and y-intercept enough information to sketch the teacher's par			134	3	A								
17	What's wrong with my graph and how can I make it better?			234	3A									
18	Why is the y-intercept not the vertex of this parabola?				13A									

Figure 3. Coding of Mrs. W’s Questions during the first three tasks.

DIMENSIONS OF MATHEMATICAL STORIES EVIDENT IN DESIGN

In this section, we briefly highlight four dimensions of differences in the ways these enacted tasks were designed: their story elements, theme, plot, and moral.

Story Elements (Characters, Actions, Settings). Both teachers selected the points used in their enactments. In Mr. J’s task design, intercepts given in the first and second parabolas were again used as the intercepts for the third pair, while Mrs. W’s intercepts for the third parabola were unrelated to the intercepts of the first two parabolas. Mr. J’s task design enabled relationships to exist between the mathematical characters (the first two parabolas shared characteristics with the third) while Mrs. Wilson’s did not.

Theme. Interestingly the theme, or what the story was about, also differed. Mr. J's story focused on sketching parabolas based on given pieces of information and considering the range of parabolas possible with that information. In contrast, Mrs. W's story involved trying to figure out a specific parabola based on partial (and sometimes insufficient) information. Thus, the theme of Mr. J's story was a *tour of parabolas* while that of Mrs. W's was a *quest for particular parabolas*.

Plot. As the diagrams in Figures 2 and 3 demonstrate, the way the questions emerged and were answered throughout this enactment differed. Although both enacted task designs contained 18 questions, most of Mr. J's questions were resolved soon after they initially appeared. In contrast, Mrs. W's mathematical story kept questions open longer, withholding resolution until all three parabolas had been "guessed." The enacted task design of Mrs. W enabled suspense, as evident from the multiple open story arcs throughout and the student reactions ("wait…" and "woo!").

Moral. Finally, the enacted designs had different mathematical morals. Mr. J's design of the task highlighted that when the three intercepts of a parabola are known, everyone will produce the same parabola through those points. The moral of Mrs. W's enacted task design was that it takes three pieces of information about a parabola (e.g. the intercepts) to be able to reliably guess it.

DISCUSSION

The two teachers described in this study were both influenced by the same written text, but each purposefully made different decisions about the stories that were "told" in their own classrooms. While one might be tempted to say that both teachers "did" this written task, we find that when comparing the two enactments, there is enough variation in their characters, themes, morals and plots to question whether they did the same mathematics at all. Examining enacted task design in this way is an important component of understanding the powerful role that teachers play in creating meaningful mathematical classroom experiences.

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