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The measurement of birefringence in optical glass by electronic means

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THE MEASUREMENT OF BIREFRINGENCE IN OPTICAL GLASS BY ELECTRONIC MEANS

by

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

INTRODUCTION

This paper presents the results of research and study into the problem of determining a practical, yet accurate method of measuring quickly the amount of birefringence (or optical strain) present in plane parallel blanks of optical glass which are to be ground and polished into lenses. The latter are often quite large and expensive, and their acceptance or rejection for grinding and polishing depends to a substantial degree on the extent of this strain.

This strain can be estimated qualitatively by merely observing the nature of the shadow-pattern between crossed nicols or Polaroid sheets. Quantitative results of higher precision, especially in border-line cases are, however, often required. The results can be expressed as difference of index in the two principal directions, or as optical path difference, or phase difference or retardation per centimeter of thickness, or as total O.P.D. or total phase difference.

In general, methods of measuring birefringence in optical glass can be divided into two principal groups. The first, and better known, group consists of so-called "compensators", which are devices for adding calibrated optical thickness of known birefringence to compensate or neutralize the birefringence of the specimen. The amount of birefringence introduced is a measure of the specimen's own birefringence. The second group consists of devices for determining the extent of ellipticity of the polarized light transmitted by the specimen. This is usually accomplished by measuring the resultant light intensities in specific directions or planes through the analyzer. Some methods from each group, both established and new will be described briefly in this introduction along with reasons for rejecting them as the adopted method. The adopted method will be described, explained, and evaluated in considerable detail in the main body of the thesis.

SOME ESTABLISHED METHODS

A. COMPENSATORS

By far the most practical methods of the first group are those using Babinet or Soleil compensators. These, like most methods, can be used with either circular or plane-polarized light. One great advantage of circularly polarized light is that it eliminates the need of aligning the specimen with the polarizer. A second advantage is that it eliminates the characteristic "cross" of dark areas seen with plane-polarized light in cases of radial symmetry.

1. Babinet Compensator

The Babinet Compensator consists of two highly accurate thin quartz wedges cut so as to oppose fast and slow axes. By sliding these wedges over each other, any necessary phase or path difference can be obtained. Since the optical thickness varies along the compensator, we see alternate light and dark bands when viewing with the compensator between crossed plane-polarized plates and using monochromatic light. With white light we see its colored components repeated in place of the light and dark bands.

A method of using the Babinet Compensator follows:

1. The compensator is placed between plane polarizer and plane analyzer, and an optical system is focussed on one of the dark bands.

2. The specimen is placed between polarizer and compensator, causing displacement of the band.

3. The compensator is adjusted by a set-screw to return displaced line to its original position. The retardation is then read off directly.

2. <u>Soleil Compensator</u>

The Soleil Compensator is similar to the Babinet but has the two quartz wedges with similar axes aligned and a rectangular slab of oppositely aligned quartz cemented beneath the second wedge. The two thin wedges move over each other by means of a finely threaded set-screw.

Use like Babinet or as follows:

1. Compensator is placed between circular polarizer and plane analyzer (at 45°). (Intensity uniform in all azimuth positions).

2. Specimen is inserted between circular polarizer and compensator, and is lined up with compensator, "fast" axis to "fast" axis. Intensity is no longer uniform in all azimuths of analyzer.

3. Compensator is adjusted by set-screw to return intensity to uniform in all azimuths of analyzer. Read off O.P.D.

Both Babinet and Soleil Compensators can employ either white or monochromatic light.

The Soleil Compensator has the great advantage over the Babinet of having uniform optical path difference over its whole area, whereas the Babinet can be used at each adjustment at only one single line position of its surface, varying in thickness at every other position.

3. Graduated Quartz Wedge

The graduated quartz wedge consists of two thin quartz wedges cemented together with opposite axes aligned and the resultant optical path differences carefully marked off along the wedge. It is used, usually in a specially-built microscope, in a manner similar to that of the Babinet compensator except that the birefringence may be read off directly without set-screw adjustment by observing the position of the dark band corresponding to the position where the wedge's optical path difference just neutralizes that of the specimen. The crossed nicols cause extinction at that point.

All three methods have the same objection for the measurement of birefringence in large blanks, namely, that they require lining up of specimen and wedge. The lining up of the specimen means that heavy, bulky glass blanks will have to be rotated about each point under test, and this presents a mechanical problem.

B. INTENSITY-MEASURING DEVICES

1. Mendenhall, Ingersoll, and Johnson Method

(Using method patented by J. T. Littleton; U.S. Patent 1,681,991 (1928)

The method outlined below, established in 1927, employs white light or infra-red with thermopile or photo-cell. The method will be described in some detail since it is similar to that proposed by the writer. Obviously, the thermopile responds only to the infra-red accompanying the white light, and the phase difference determined is for such infra-red. (Both photo-cell and thermopile require a suitable galvanometer).

The source of energy is a "pointolite" lamp. The light converges at the specimen to a circular spot about four millimeters in diameter. Some advantages of non-parallel light (in spite of some difficulties) are:

1. A small area is under investigation at one time.
2. Small surface irregularities have little disturbing effect, as the beam diverges again from its focus at the surface of the specimen.

Method of Operation:

The beam is passed through crossed polarizers, converging between them. Partly due to non-parallel light it is impossible to reduce the minimum below about 0.1% of the incident energy.

The specimen is then placed between the crossed polarizers at the point of convergence, and the galvanometer deflection is noted. The specimen is mounted in a special holder and rotated about the axis of the beam. The galvanometer deflection passes through a maximum and minimum 45° apart. From the maximum, corrected as below, the depolarizing action of the specimen and, therefore, its birefringence may be calculated.

Theory:

Plane-polarized light of azimuth 45° (from the first nicol) is incident on the specimen, whose principal directions are lined up horizontally and vertically. The birefringence causes a phase difference of the two components of vibration (vertical and horizontal) on leaving the specimen, resulting in elliptical vibration. The square of the component of vibration at right-angles to the original azimuth of polarization, i.e., in the plane of the analyzer, determines the galvanometer deflection D.

D' equals the deflection corresponding to the combined energy of the two components giving rise to the ellipse. The most direct way to obtain this is to insert the specimen parallel in axes to the principal planes of the nicols, and add the deflections. Then $(D/D')^{\frac{1}{2}}$ is a measure of the phase difference. For large ellipticities a simple calculation is needed.

The method is applicable to phase differences less than 90°. The phase difference per centimeter is taken as a measure of the strain.

(Insert above:- The net deflection obtained by subtracting the minimum from the maximum gives D).

Principal objections:--Specimen must be lined up with the principal directions of the polarizers and must be rotated about each point examined. Also, the method is limited to phase differences less than 90°. In thick optical blanks higher phase differences are often found.

Lining up big, heavy optical blanks in the special holder described would be very difficult. As already indicated, special equipment for this purpose might be expensive, difficult, and bulky.

SOME NEW METHODS

1. "Steps" of retardation

This suggested method consists of employing predetermined "steps" of varying mica thickness, as outlined below. The method is not new in principal but is included here only because it was the first to be given serious consideration. While the outlined method employs monochromatic light, this "step" compensator can be used in almost every way employed with the Babinet or Soleil compensators.

The method follows:

1. There are steps of varying mica thickness ($\Delta\mu = .034$), for example, ten equal steps from 0 to $\lambda/2$, and an inter-step "fine" series of five steps from 0 to $\lambda/20$, i.e. ($\lambda/100$, $\lambda/50$, $3\lambda/100$, $\lambda/25$, $\lambda/20$).
2. The specimen is placed between a circular polarizer and a

plane analyzer. Step compensator is between specimen and analyzer. The "fast" axis of the compensator is lined up with the "slow" axis of the specimen.

3. Add steps and intersteps until analyzer shows uniform intensity in all azimuths. The sum of the steps equals the path or phase difference of the specimen.

Principal objections:--Specimen must be lined up with compensator. Also, adding up steps is a nuisance, especially since slight differences in light intensity are hard to appreciate visually. (Weber-Fechner Law states that visual intensity (like other sensory intensities) increases as the logarithm of the stimulus, except at the extremes of the intensity range).

2. "Oblique-path" compensator

This second suggested method makes use of oblique paths through a quarter-wave plate to obtain varying optical thicknesses. (Other, thicker than quarter-wave plates could be used equally well with slight change in procedure). Although mica can be used, quartz would be better, for mica is slightly biaxial and may introduce a third index due to the oblique nature of the path. As for the separating of the E and O rays (or double-image) that results, this is of no consequence if we have coherent parallel light (from a small source, through a collimating lens). Thus, for every separation, there is a new overlapping with the same phase difference maintained.

The method follows:

1. Plane-polarized monochromatic parallel light is sent through a quarter-wave plate and the specimen, the plane-polarizer being orientated at 45° to the specimen's principal planes.

2. Rotate quarter-wave plate about optical axis of the plate (which is lined-up with its fast axis opposite the slow axis of the specimen), until the intensity is uniform for all azimuths of the analyzer.

3. Divide the sine of the angle of rotation by the index in that plane. This gives sin R (the angle of refraction). Determine cos R. Divide quarter-wave plate thickness by cos R to get new thickness. Actually, $90^\circ/\cos R$ will give the new total phase difference, since phase difference is proportional to thickness in uniformly birefringent media. For the birefringence of the specimen, just subtract the 90° of the quarter-wave plate. Thus:

$$90^\circ/\cos R - 90^\circ = \text{birefringence of specimen.}$$

Other thicknesses of the plate besides quarter-wave may be used with minor changes in the procedure.

Photo-cell or thermopile methods may also be used. When the analyzer is rotated rapidly, any lack of uniformity in intensity can be made to actuate an amplifier and oscilloscope. The lack of uniformity will show on the oscilloscope as a vertical spread of the trace, if the photo-cell connects to the vertical (Y) input. The trace may be controlled and timed horizontally by rotating a potentiometer along with the analyzer. Of course, a motor must be used to do the rotating.

Principal objections:-Specimen must be lined up with quarter-wave plate and polarizer (45°). Visual estimation of uniform intensity is difficult. Oblique rotation of plate requires complex device. (Limited to fairly small amounts of birefringence, since large angles of incidence will introduce errors through reflection losses.)

THE METHOD FINALLY CHOSEN

The method finally chosen combines some of the best features of methods outlined above with new ideas, and eliminates the principal objections indicated above. While far from ideal, it is particularly adapted to large optical glass blanks, which is essentially the problem we are interested in. It is a radiation or intensity-measuring device, and is discussed fully in the material that follows.

DESCRIPTION OF THE ADOPTED METHOD.

Basically, the method consists of sending a beam of collimated monochromatic light through a circular polarizer, then through the specimen whose birefringence is to be measured. The resultant elliptically polarized light is then analyzed by passing the beam emerging from the specimen through a plane analyzer (e.g., a nicol prism or Polaroid plate) which may be rotated to positions of maximum and minimum intensity. These light energy intensities are measured by means of a photo-electric cell. The section on Theory (to be considered at length presently) will show by the results of mathematical investigation of elliptic light, that if the maximum reading on the ammeter is A, and the minimum reading B, then the total phase difference of the specimen equals the angle whose sine is the quotient obtained by dividing the difference, A - B , by the sum, A+B, i.e.,

$$\Delta \phi = \sin^{-1}\left(\frac{A-B}{A+B}\right) = \sin^{-1}\left(\frac{I_{MAX}-I_{MIN}}{I_{MAX}+I_{MIN}}\right)$$

The source used is a six-volt tungsten lamp, controlled through a transformer and "variac." A voltage stabilizing device in the line current or in the instrument is strongly recommended. Light from the lamp is focussed onto a pinhole, and is collimated into a parallel beam by a spherically corrected lens with an aperture 15 mm. in diameter. This collimated beam is next sent through a circular polarizer consisting of a plane Polaroid filter to which has been cemented a quarter-wave plate, the direction of polarization being at 45° to a principal direction of the plate. This circular polarizer can be purchased ready for us (Polaroid). It is important that the light be incident on the plane polarizing portion first, then on the quarter-wave plate.

On leaving the quarter-wave plate, the collimated beam is, of course, circularly polarized only for one wavelength, usually yellow-green light at 550 millimicrons. Light of other wave lengths is eliminated by a yellow-green monochromatic filter (550 millimicrons).

Circular polarization of the beam has the great advantage of permitting the emergent beam to enter the specimen in any azimuth of the principal directions of the specimen with equal effect. This means no lining up of the specimen is necessary, the only requirement being normal incidence of the light onto the specimen, assuming the optic axis lies parallel to the surface of the specimen. This advantage of freedom from lining-up is due to the fact that circularly polarized light is mathematically equivalent to two plane-polarized beams of equal amplitudes, which are vibrating at right angles (i.e., in planes perpendicular to each other), orientated in any arbitrary azimuths chosen, provided only that the phase difference is 90°. Thus, we can imagine that the specimen is really lined up with the principal directions of the quarter-wave plate, and we can treat the total birefringence that results, as if it were due to one continuous thick plate consisting of two parts of different media, since the quarter-wave plate is always as if lined up with the specimen, regardless of the position in which the specimen is inserted.

A second advantage obtained by employing circularly polarized light is the elimination of the "cross" commonly obtained in circular glass blanks which are viewed between plane-polarizer and plane-analyzer. This simplifies the theory and also permits simpler visual justification and comparison with results obtained by the instrument, since the eye then sees what the instrument "sees."

Actually from this point on we may consider the total birefringence in terms of phase difference produced by the combination of the quarter-wave plate and specimen, as if it were produced by a single medium. As the section on theory will show, the resultant state of polarization of the emergent beam from this combination is elliptical and may be represented as an ellipse except for total phase differences of odd multiples of $\pi/2$, when the ellipse becomes a circle, and total phase differences of even multiples of $\pi/2$, when it degenerates to a straight line. The lengths of the semi-axes of the ellipses, or of the radius of the circle, or the half-length of the straight line representing the state of polarization are measures of the relative amplitudes in their respective azimuths. The intensities are therefore determined by the squares of these lengths. The maximum and minimum intensities are thus proportional to the squares of the semi-major and semi-minor axes of the resultant ellipse, since these are the maximum and minimum axes of the ellipse. (Circles and lines may be considered special and limiting forms of the ellipse).

In order to justify the use of the plane-analyzer, it is necessary to show that any of these ellipses representing, as Lissajous figures, the state of elliptic polarization resulting from two plane-polarized waves vibrating in mutually perpendicular planes and with equal amplitudes at any given phase difference, will also result from two plane-polarized waves of unequal amplitude vibrating in mutually perpendicular planes containing the semi-major and semi-minor axes of the ellipse respectively but at 90° phase difference only. This is readily shown in the section on mathematical theory to be considered presently. Hence, the components of the beam vibrating in the planes of the major and minor axes of the ellipse may be considered to be plane-polarized and may be readily analyzed by the plane-analyzer. Therefore, we now pass this elliptically polarized beam through a rotatable plane-polarized analyzer (Polaroid plate). When this plane-analyzer is in the azimuth of the major axis, all the light vibrating with maximum amplitude and therefore with maximum intensity will be transmitted, and when this plane-analyzer is in the azimuth of the minor axis, all the light vibrating with minimum amplitude and intensity will be transmitted. The maximum and minimum intensities can then be determined by allowing this light to be focussed onto a photo-cell and the corresponding intensities measured, as has been previously indicated. In actual use, it is merely necessary to rotate the analyzer by hand and note the maximum and minimum values of intensity.

To save time in usage, the meter scale can be modified to read the birefringence directly in terms of phase-difference. This can be accomplished most easily by adjusting the intensity of the source (once the "maximum" position of the analyzer has been located) to give a constant "maximum" intensity reading, A , on the ammeter. This constant was arbitrarily chosen at some mid-scale point (e.g., "60"), but could be chosen as "100" or any other point in the upper portion of the scale. (N.B: The reason for the writer's preferring a point not too near the end of the scale is mainly that the values are crowded there and therefore a bit more difficult to read quickly. In addition, certain possible techniques using circular analyzers require room for recording maxima at points above this constant "maximum" which will be designated on the scale as "MAX"). Once the maximum reading, A, has been determined and relocated at the constant position "MAX", the minimum intensity, B, is then determined and the birefringence in terms of phase difference can be read off directly from the position of the needle at this minimum "B" point on a superimposed scale which has been worked out previously from the expression $\sin(\Delta\phi) = \frac{A-B}{A+B}$, where "A" is given the value of the constant scale point above, (MAX).

To show that $\sin(\Delta\phi)$ depends merely on the ratio between B and A, (i.e., B/A), and not on the absolute values of A and B, it is simply necessary to divide the numerator and denominator of the expression $\frac{A-B}{A+B}$ by A. The expression then becomes $\frac{1 - B/A}{1 + B/A}$ without changing the value of $\sin(\Delta\phi)$.

A scale drawing of a modified chart similar to the one described above is shown on the following page. This particular drawing is made to be used in conjunction with the regular meter dial, and is not intended to be superimposed over it.

GALVANOMETER	0	2.5	5	10	15	20	25	30	35	40	45	50	55	60 (MAX)	65	70
UPPER DIAL	270°	293°	302°	314.5°	323°	330°	336°	340.5°	345°	348.5°	352°	355°	357.5°	360°		
		247°	238°	225.5°	217°	210°	204°	199.5°	195°	191.5°	188°	185°	182.5°	180°		
LOWER DIAL	90°	113°	122°	134.5°	143°	150°	156°	160.5°	165°	168.5°	172°	175°	177.5°	180°		
		67°	58°	45.5°	37°	30°	24°	19.5°	15°	11.5°	8°	5°	2.5°	0° MAX		

SPECIAL SCALE FOR READING BIREFRINGENCE DIRECTLY

Locating the Principal Directions of the Specimen

While not essential to the purpose of the instrument, it enhances its value as a laboratory "tool", if we can use it to determine quickly and fairly accurately the principal directions (i.e., the direction of the optic axis and the direction at right angles to it, both assumed to be in the plane of surface of the optical blank) of the specimen. This can be done by determining the necessary position of the analyzer at maximum intensity with respect to the fast axis of the specimen.

Between 0° and 180° of phase difference, the maximum intensity position of the analyzer occurs 45° counter-clockwise of the specimen's fast axis, if the original polarizer position is 45° clockwise of the fast axis of the quarter-wave plate which together with this original plane-polarizer makes up the fixed circular polarizer of the instrument. (Between 180° and 360° of phase-difference, the maximum position of the analyzer occurs 45° clockwise of the fast axis of the specimen; but this amount of phase difference rarely occurs in practical usage, since qualitative rejection of optical blanks occurs before 180° of phase difference is reached, with a few exceptions).

While 180° of phase-difference cannot be readily distinguished from 0° by this device in uniform specimens, i.e., by the elementary device that has been described up to this point in the paper,(for a method of differentiating these two conditions will be described later), specimens of optical blanks invariably show gradations of birefringence which offer a substantial clue as to whether we are dealing with near-0° of phase-difference or near-180° of phase difference. In this connection it may be stated at this time that due to the nature of the annealing, the normal strain pattern in circular optical blanks arranges itself with one principal axis along the radius and the other along the tangential direction. Thus, at the center of the circular blank where all directions are radial, the birefringence should be 0° or very near 0° . Then, as we progress along any radius, the birefringence should gradually increase. These intermediate points could easily be measured or spotted qualitatively, and, thus, there need never be any difficulty in distinguishing between near-0° and near-180° of phase-difference.

This change in position of the fast axis of the specimen (described above) relative to the position of the analyzer depending on whether we are dealing with less than 180° or more than 180° (up to 360°) of phase-difference may well be used to determine whether we are dealing with less than one-half wave or more than one-half wave of birefringence (up to a full wave) and, in fact, a method of doing this will be described later. As of now, it suffices to say that the position of the fast axis in the specimen can be determined previously by other methods and thus can be used to distinguish between the two divisions of phase difference (as indicated above).

Mathematical Theory

The birefringence theory applicable to our method is that involved in determining a mathematical expression for the phase difference suffered by two plane-polarized plane-waves polarized perpendicularly to each other, starting out simultaneously, and travelling the same identical straight path through a uniaxial crystal in a direction perpendicular to the optic axis. First, this expression is to be determined for two waves of different amplitudes, then for waves of equal amplitudes. It is the latter case that primarily concerns us. When they start out, these two waves of equal amplitude may be expressed as the components of a single plane-wave plane-polarized in a resultant azimuth half-way between the two equal components (i.e., at 45°). Mathematical analysis of the expression for the phase difference (below) will show that at 90° (or one-quarter wave-length) of phase difference, the two components form a resultant which rotates at uniform velocity and is of constant amplitude. This resultant is therefore a circularly polarized wave. Whether it is rotating clockwise or counter-clockwise depends upon the position of the optic axis relative to the direction of polarization of the original plane-polarized incident wave, i.e., whether it is 45° clockwise or 45° counter-clockwise of this direction.

At 180° of phase difference the two components again form a resultant single plane-polarized wave just as at incidence but in a direction of vibration (polarization) perpendicular to that at incidence. Between these 0° and 180°

of phase difference, the resultant of the two perpendicular components at any point is also rotating, but with varying amplitudes, the amplitudes at various angles of rotation being expressible as the magnitudes of the semi-axes of an ellipse whose semi-axis major is parallel to the incident beam's direction of polarization for phase differences between 0° and 90°, and perpendicular to its direction for phase differences between 90° and 180°. At 270° of phase difference we again have circularly polarized light, but now rotating in the opposite direction from that at 90°. At 360° of phase difference we have exactly the same condition as at incidence, i.e., the two components are just one wave-length apart and, therefore, the amplitudes at crest just coincide to form the original incident plane-polarized wave again and in the same azimuth.

Mathematical Theory (continued):--The addition of Two Simple Periodic Motions at Right Angles to Find the Resultant Motion. (Standard Proof)[8]

Same frequency; displacement in two perpendicular directions impressed simultaneously on a point:-

$$y = r_1 \sin (wt + \alpha_1)$$

$$z = r_2 \sin (wt + \alpha_2)$$

Add (superimpose) to find path of resultant motion:

$$(1) \quad \frac{y}{r_1} = \sin wt \cos \alpha_1 + \cos wt \sin \alpha_1$$

$$(2) \quad \frac{z}{r_2} = \sin wt \cos \alpha_2 + \cos wt \sin \alpha_2$$

We must eliminate t.

Multiply (1) by sin α_2 and (2) by sin α_1; then subtract (2) from (1):

$$(3)\quad \frac{y}{r_1}\sin\alpha_2 - \frac{z}{r_2}\sin\alpha_1 = \sin\omega t\,(\cos\alpha_1\sin\alpha_2 - \cos\alpha_2\sin\alpha_1)$$

Similarly, multiply (1) by cos α_2 and (2) by cos α_1; and subtract (1) from (2):

$$(4)\quad -\frac{y}{r_1}\cos\alpha_2 + \frac{z}{r_2}\cos\alpha_1 = \cos\omega t\,(\cos\alpha_1\sin\alpha_2 - \cos\alpha_2\sin\alpha_1)$$

Now eliminate t by squaring and adding (3) and (4):

$$(5)\quad \sin^2(\alpha_2-\alpha_1) = \frac{y^2}{r_1^2} + \frac{z^2}{r_2^2} - \frac{2yz}{r_1 r_2}\cos(\alpha_2-\alpha_1)$$

Equation of resultant path.

We can now graph this equation of resultant path for various values of the phase-difference $\Delta\alpha = \alpha_2 - \alpha_1$. These curves are all ellipses except for the special cases where they degenerate into straight lines. The major and minor axes of the ellipse are inclined to the y and z axes except when $\Delta\alpha = \pi/2, 3\pi/2, 5\pi/2, \ldots$, when they coincide with them. In respect to this precise equation of path, we are only concerned with the first of these exceptions, i.e., what the path is when $\Delta\alpha = \pi/2$. Substituting in the equation of path (above), we obtain in this case:

$$(5a)\quad \frac{y^2}{r_1^2} + \frac{z^2}{r_2^2} = 1$$

This is the equation of an ellipse with semi-axes r_1 and r_2 coinciding with the y and z axes, respectively. This proves that two plane-polarized waves vibrating in mutually perpendicular planes with unequal amplitudes, r_1 and r_2 respectively, and with phase difference always 90°, will have a resultant motion whose path will be an ellipse with semi-axes, r_1 and r_2, lying in the perpendicular planes of vibration. Such an ellipse can always be considered, then, to be made up of the original two plane-polarized waves, and can be plane-analyzed accordingly.

What, then, is the resultant path where we are dealing with perpendicular plane-waves of equal amplitudes, as in our device, i.e., where $r_1 = r_2 = r$? Simply substitute r for r_1 and r_2 , in equation (5) above, and we obtain the equation of the resultant path for our case:

$$(6) \quad \sin^2(\alpha_2-\alpha_1) = \frac{y^2}{r^2} + \frac{z^2}{r^2} - \frac{2yz}{r^2}\cos(\alpha_2-\alpha_1)$$

or, $$(7) \quad r^2 \sin^2\Delta\alpha = y^2 + z^2 - 2yz\cos\Delta\alpha$$

Except for the special cases where they degenerate into straight lines, the graphs of this equation (7) are all ellipses (or circles) for the various values of $\Delta\alpha$. They are all inclined (semi-axes) 45° to the y and z axes except for phase differences equal to odd-multiples of $\pi/2$, as above. In our case, these are circles with r equal to the radius, and the light emerging with these phase differences is circularly polarized, and may be considered to be the resultant of two plane-polarized waves vibrating in any two mutually perpendicular directions, and can be plane-analyzed accordingly. It is well to note that the direction of rotation of the resultant radius vector changes from that at $\pi/2$, when we reach a phase difference of $3\pi/2$, and changes back again at $5\pi/2$.

The maximum and minimum values of the resultant amplitude vectors form the semi-major and semi-minor axes respectively of these ellipses. To make these principal axes coincide with new y and z directions (for simplicity's sake), a simple orthogonal change of coordinates suffices, i.e., a rotation of axes. The transformation formulae for such a change from 45° and 135° to 90° and 180° are as follows:

To rotate axes 45° :

$$y = y' \cos 45° - z' \sin 45°$$
$$z = y' \sin 45° - z' \cos 45°$$

Therefore, $y = \frac{y'-z'}{\sqrt{2}}$; $z = \frac{y'+z'}{\sqrt{2}}$.

Substituting,

$$r^2 \sin^2 \Delta\alpha = y^2 + z^2 - (y^2 - z^2)\cos\Delta\alpha$$
$$= (y^2+z^2)(1-\cos\Delta\alpha) + 2z^2\cos\Delta\alpha$$
$$= y^2(1-\cos\Delta\alpha) + z^2(1+\cos\Delta\alpha)$$

where all primes have been dropped, but are "understood."

Now, divide by $r^2 \sin^2 \Delta\alpha$ and rearrange coefficients of y^2 and z^2:

$$(8) \qquad 1 = \frac{y^2}{\frac{r^2 \sin^2\Delta\alpha}{1-\cos\Delta\alpha}} + \frac{z^2}{\frac{r^2 \sin^2\Delta\alpha}{1+\cos\Delta\alpha}}$$

Let $\frac{r^2 \sin^2\Delta\alpha}{1-\cos\Delta\alpha} = a^2$, and $\frac{r^2 \sin^2\Delta\alpha}{1+\cos\Delta\alpha} = b^2$

Then, (8) becomes

$$1 = \frac{y^2}{a^2} + \frac{z^2}{b^2}$$

This ellipse is identical with that of equation (5a) with $a = r_1$, and $b = r_2$, the respective amplitudes of the two plane-polarized waves into which the elliptic light can be resolved ($\Delta\alpha = 90°$). Likewise, a and b are the semi-axes of this ellipse, major and minor, and a plane-analyzer whose direction of polarization coincides with either a or b will transmit all the light plane-polarized in that direction. The intensity of this transmitted plane-polarized light will be a^2 and b^2 respectively. Now, solving the identities at the top of this page for $\cos\Delta\alpha$ in terms of a^2 and b^2, we find first that

$$a^2-b^2 = r^2\sin^2\Delta\alpha\left(\frac{2\cos\Delta\alpha}{1-\cos^2\Delta\alpha}\right), \text{ and}$$

$$a^2+b^2 = r^2\sin^2\Delta\alpha\left(\frac{2}{1-\cos^2\Delta\alpha}\right),$$

and, finally, $$\frac{a^2-b^2}{a^2+b^2} = \cos\Delta\alpha = \frac{I_{MAX}-I_{MIN}}{I_{MAX}+I_{MIN}}$$

or, letting $a^2 = A$, and $b^2 = B$,

$$(9) \quad \cos\Delta\alpha = \frac{A-B}{A+B}.$$

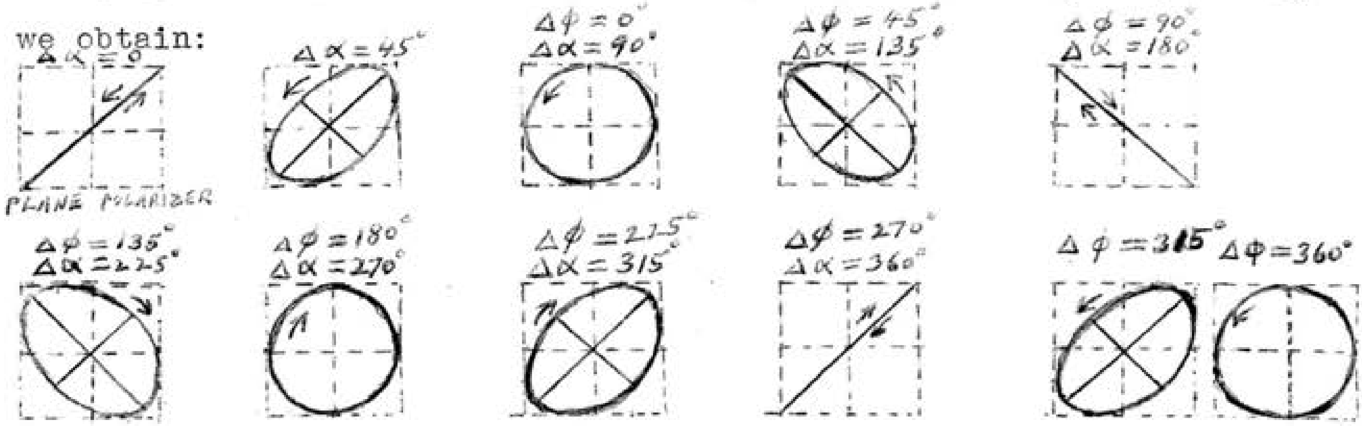
Therefore, we have proved that we may rotate a plane-analyzer in the path of a beam of elliptically polarized light, and determine the cosine of the phase difference or the birefringence of the media causing the ellipticity in terms of the maximum and minimum intensities of the light transmitted by this plane-analyzer, as in equation (9).

We now have the cosine of the phase difference due to the entire birefringent medium lying between the plane-polarizer and the plane-analyzer. However, 90° of this phase difference is due to the quarter-wave plate cemented to the plane-polarizer, and the remainder ($\Delta\alpha - 90^\circ$) is due to the specimen under test. Therefore, since $\sin(\Delta\alpha - 90^\circ) = \cos\Delta\alpha$, and letting $(\Delta\alpha - 90^\circ) = \Delta\phi$,

$$\boxed{\frac{A-B}{A+B} = \cos\Delta\alpha = \sin\Delta\phi,}$$

and we have finally arrived at the equation used on our device to determine the birefringence of the specimen in terms of maximum and minimum intensities.

If we now interpret the Lissajous figures,[8] which are the graphs of equation (6) above, in terms of angle $\Delta\phi = (\Delta\alpha - 90^\circ)$, we obtain:

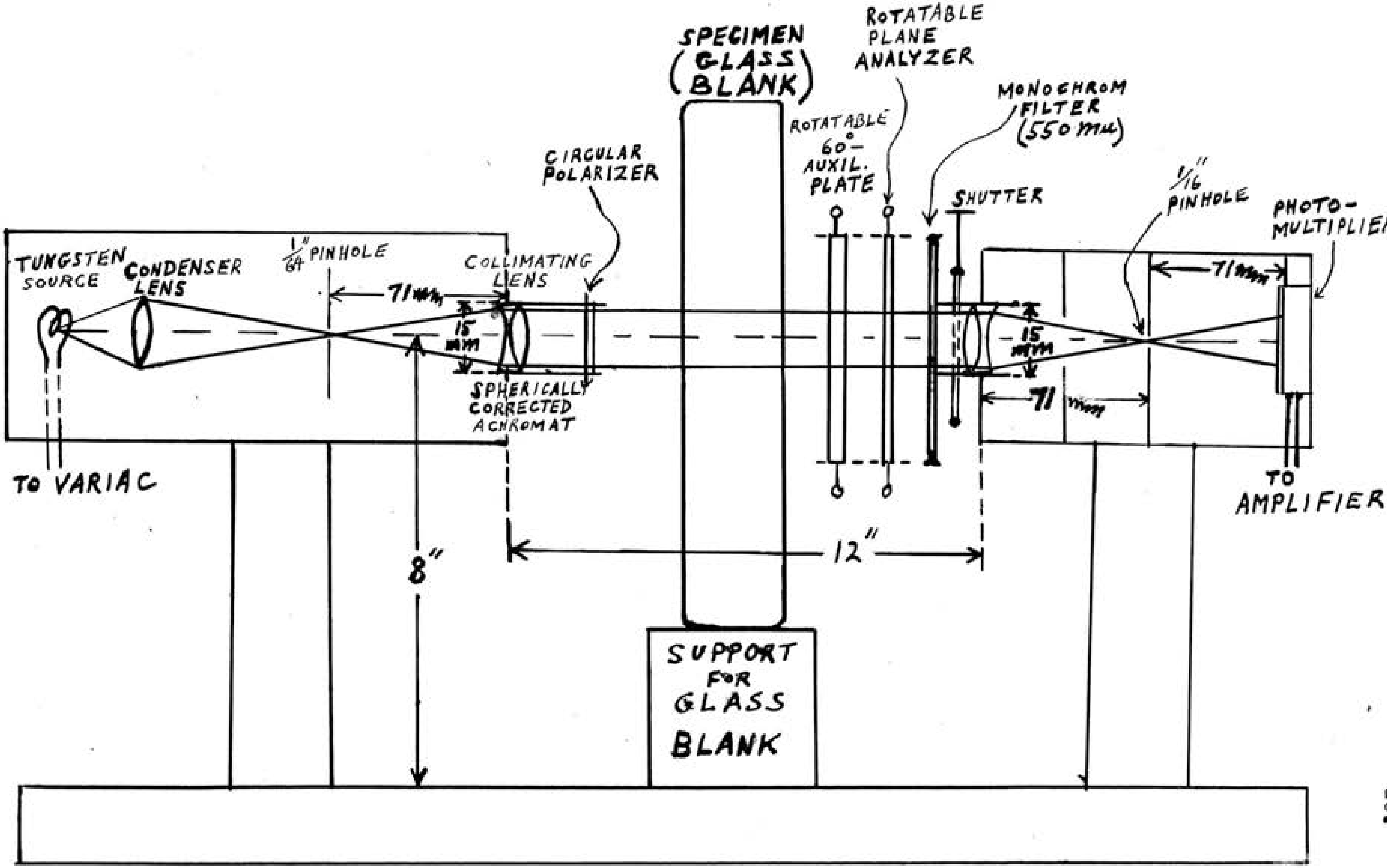


The drawings above show very clearly that phase differences of 0° and 360° give identical figures with same rotation sense, and can't be distinguished by the device alone. Similarly, 45°, 135°, 225°, and 315° can only be distinguished by rotation sense or by direction of analyzer at maximum in relation to plane-polarizer. Methods for utilizing these facts will be discussed.

Optical and Mechanical System:

The optical system of the instrument is best described by reference to the schematic diagram on the following page. The diagram also shows schematically the mechanical and positional requirements of the instrument.

Light from the tungsten-lamp source is focussed by its condenser onto a pinhole of about 1/64" diameter. Light from this pinhole diverges to a spherically corrected achromat of 15 millimeters diameter, which is placed at its focal length from the pinhole. The beam of light transmitted by this lens is thus collimated accurately and then allowed to pass through a circular polarizer, the position of whose incident plane of polarization in respect to the fast axis of its quarter-wave plate is known to be either right or left and marked accordingly. The circularly polarized collimated beam is then allowed to penetrate at normal incidence the specimen under test. Room is left for several possible auxiliary plates or analyzers which are rotatable and may be inserted or removed from the beam at will. The beam now passes through a rotatable plane-analyzer, whose direction of polarization (Polaroid) is plainly marked on the edge of the filter, with degree-marks on the edge of the holder (ring). The beam then passes through a yellow-green filter (550 millimicrons), and through a final lens, a spherically corrected achromat of focal-length 71 millimeters as before (both should be coated), whence it is focussed onto a 1/16" pinhole enclosed in a light-proof box whose only admission of light is through the lens just described. The pinhole is, of course, located at the focal point of this lens, and the beam converging to it is also required to pass through a circular "baffle" of diameter less than the 15 millimeter diameter of the lens. From the pinhole the beam diverges within the light-proof box to a photo-multiplier or photo-cell within the box. The final lens is provided with a shutter.



OPTICAL SYSTEM SHOWING POSITIONAL AND MECHANICAL DATA

Up to this point, we have a simple device that has essentially all the advantages of the Mendenhall, Ingersoll, and Johnson device described in the introduction without many of its objectionable features. The principal advantage of our method is that the specimen does not have to be lined up nor rotated about each point examined. Our device requires no special skill in its simple form and can be readily used by a novice. However, we are still limited at this point to measuring phase differences between 0° and 90°. Much of the remaining discussion will concern methods of correcting this limitation.

60° - Auxiliary Plate.

A rotatable 60°-phase-difference plate (made up from a uniaxial crystal, or mica, or even cellophane) is mounted between specimen and analyzer in a manner such that it may be inserted into or removed from the collimated beam at will. Thus, the auxiliary plate may be thrust aside for ordinary usage and pushed back into the path of the beam whenever necessary. The fast-axis of the plate must be marked. The use of this auxiliary plate will be explained below in discussing recommended procedure.

RECOMMENDED PROCEDURE IN ROUTINE TESTING:

Since this instrument is primarily intended for measuring birefringence in optical blanks, the procedure recommended will be formulated with this primary usage in mind.

Optical blanks or similar specimens should be plane polished or quite smooth on both sides. (It is possible to obtain an <u>informative</u> reading with unpolished blanks ("rippled" lines, etc.), but this reading will probably not be accurate enough for quantitative usage). Also, the specimen may be inserted unpolished in a parallel-sided glass tank, and the immersion method used with proper mixture of oils of different indices. In most cases it will be just as simple to polish the blank.

A. <u>Small Birefringence</u>

Step 1.

Observe optical blank between crossed Polaroids. Points of maximum density distortion are then circled for quantitative investigation. Similarly, the blank may be observed between circular polarizer and plane analyzer. On rotating specimen or analyzer, the points of maximum strain will show as density distortions or variations, and may be likewise circled. The latter method has the advantage of enabling the observer to see subjectively what the instrument "sees" objectively.

Step 2.

The optical blank is now ready for quantitative measurement. It is placed in the instrument between polarizer and analyzer in a position such that the collimated beam is incident normally on its surface and passes through the center of the encircled area to be measured for optical strain. The analyzer is now rotated to a maximum intensity position.

Step 3.

The "variac" is now adjusted until the maximum intensity is indicated on the meter's scale at the chosen constant point (to be marked above the 0° (180°) mark with letters MAX).

Step 4.

The analyzer is now rotated to a minimum intensity position. In this position the meter needle will indicate the birefringence measurement directly in terms of phase difference (degrees).

If the indicated birefringence is 0° to 60° or even 70°, and it is known by qualitative observation that the reading lies between 0° and 90°, then the measurement is completed, and the next circled spot is moved into the beam's path for measurement, until all encircled spots are measured.

However, in thick optical flats the birefringence may be much higher than 70° or even 90°. Let us first consider the 60° to 90° region. Between 60° and 90° (also between 90° and 120°) the value of the sine varies only between 0.866 and 1.000 (span of 0.134), while between 30° and 60°, the same angular span, the value of the sine varies between 0.500 and 0.866 (span of 0.366) or almost three times as much. Thus, the instrument is relatively insensitive to the region around 90°, and fairly large angular errors in results may be expected here. To offset this expected error in the region around 90°, use is made of the 60° -auxiliary plate. The procedure follows:

B. <u>Modification to include $\Delta\phi$ up to 90°</u>. (This requires use of the 60° auxiliary plate.)

<u>Step 1:</u>

After completing basic procedure above, return analyzer to maximum position (i.e., to constant MAX mark on scale).

<u>Step 2:</u>

Insert auxiliary plate in beam, and rotate it to position 45° counter-clockwise to position of analyzer in Step 1. The auxiliary plate should now be lined up with the specimen, fast axis against fast axis. To check fast axis, if doubtful of this direction on specimen, take reading near, but not at, center of flat, (i.e., Where birefringence is $< 90°$, as determined qualitatively). Fast axis will then be 45° counter-clockwise of analyzer's maximum position for the reading.

<u>Step 3:</u>

Adjust "variac" until needle points to constant MAX position again.

<u>Step 4:</u>

Rotate analyzer to minimum intensity position and read off birefringence in degrees. Subtract 60° from the result (over 90°) for true birefringence of the specimen. (For example, scale may read (50°, 130°). Subtract 60° from 130°, giving true birefringence of 70°). Thus, we avoid the region around 90°, if fair accuracy is required. It should be noted that readings are accurate enough for most work up to 70° and beyond 110° without the use of the auxiliary plate.

The above procedures are satisfactory as far as they go, but we would like a more general procedure, preferably one that would take us from 0° to 360° with reasonable accuracy. This is outlined below.

C. General Procedure (0° to 360°).

Step 1:

Observe optical blank between crossed Polaroid plates, and encircle spots showing strain.

Step 2:

Place optical blank in instrument so that polarized collimated beam is incident normally and passes through the center of the encircled area to be tested.

Step 3:

Rotate analyzer to a maximum intensity position, and adjust "variac" until needle of meter falls on fixed point of scale designated MAX.

Step 4:

Rotate analyzer to minimum intensity position. Needle of meter now indicates birefringence in units of phase difference (degrees). There will be four possible readings above the needle, and we must distinguish among them: ($\Delta\phi$; $180° - \Delta\phi$; $180° + \Delta\phi$; $360° - \Delta\phi$).

Step 5:

Return analyzer to maximum intensity position (MAX on scale).

Step 6:

To distinguish between 0° -to-180° readings and 180° -to-360° readings.

First, estimate direction of fast axis on blank at spot under test. Do this from geometry of blank and by method previously suggested. Position of analyzer in Step 5 should be 45° to one side or the other of blank's fast axis. If fast axis is counter-clockwise of analyzer position in Step 5, we are dealing with birefringence between 0° and 180° . If we find fast axis clockwise of analyzer position in Step 5, birefringence is between 180° and 360° . (0° to 180° is on the lower part of scale, 180° to 360° on upper part).

Step 7:

To distinguish between $\Delta\phi$ and (180° - $\Delta\phi$). (LOWER SCALE)

Insert 60° -auxiliary plate in beam and rotate it to position 45° counter-clockwise of position of analyzer in Step 5 (max). Adjust "variac" until needle points to MAX on scale, again.

Step 8:

Rotate analyzer to minimum intensity position and read off combined birefringence in degrees. Subtract 60° from whichever of the four possible readings will give a remainder which is, first, between 0° and 180° , and second, is either of the two lower-scale readings to be distinguished. The remainder is the true birefringence.

Step 9:

If the readings are between 180° and 360° . (Upper scale).

Follow Steps 7 and 8, but add 60° to get a sum between 180° and

360° which is one of the two upper-scale readings in dispute. This sum is the true birefringence.

Testing the Device-----Calibration Tests.

For purposes of calibration, various samples of cellophane and mica were measured for birefringence by the new device, and the results recorded. These same items were then measured for birefringence by means of the B. & L. quartz wedge and polarizing (chemical) microscope with attachment for inserting wedge and holding analyzer cap. This is essentially the same device described in the introduction. To make doubly sure that the wedge was accurate, a B.&L. quarter-wave plate was tested in the microscope with this wedge and a slight correction necessary for accuracy determined. The results were quite satisfactory, corroborating the findings of our new device quite well.

Some examples of results are presented on the next page.

SOME RESULTS OF CALIBRATION TESTS

	Specimen	Birefringence by B. & L. Wedge	Birefringence by the New Device	%Error
1.	Cellophane #1	216°	211° ±2.5°	-2.3%
2.	Cellophane #2	140°	139° ±2°	-0.7%
3.	Mica sample	45°	42° ±2°	-6.7% [See comment below.]
4.	Quarter-wave plate (B.&L.)	90°	87° ±2.5°	-3.3%

Sources of Error (Especially in less sensitive portions of scale).

The results of these calibration tests seem quite accurate (except possibly for the mica which was flaky and less uniform in thickness than the other samples), but it must be noted that they are accurate only because they have been referred to the more sensitive portion of the scale, either directly or by use of the 60° -auxiliary plate. However, the procedure would have been much simpler, had we been able to get accurate results at all points of the scale directly. It therefore becomes of considerable interest to determine just why the regions near 90° (roughly 70° to 110°), or 270° (roughly 250° to 290°) are likely to be inaccurate to an important degree (up to about one-twentieth of a wave-length).

We know from previous discussion that the primary reason for insensitivity here is the small variation in the value of the sine for a large variation in the angle in this region. However, if the degree of accuracy found by calibration were maintained even in proportion to this, we could still obtain more accurate results than the 12% to 15% error often found here. The question remains as to why this occurs.

Among the likely contributory reasons are the following:

(1) Zeroing error. Accurate zeroing of the ammeter before testing is essential to even fair accuracy. A zeroing error of only two meter divisions out of sixty will give a reading of $\frac{60 - 2}{60 \quad 2} \cong 0.92 = \sin 67$, where the reading should be $\frac{60 - 0}{60 + 0} = 1.00 = \sin 90^\circ$. Thus, two small divisions of zeroing error lead to a phase difference error of 23°. (Use of the expanded range on the amplifier might help.)

Zeroing is accomplished by means of a shutter, as indicated in diagram on page 28, and this shutter should be used at frequent intervals during testing to check zeroing.

(2) Light leakage through the Polaroid plates. This leakage of about 1% through the crossed Polaroid analyzer becomes very important in this region where the transmission at maximum may be very nearly 100%. This means that 1% of total transmission leaks through the minimum position, while practically none is added to the maximum position. (There is also some slight leakage through the polarizer). For example, at 90°, $\sin 90^\circ$

$= \frac{100-0}{100+0}$. But, with 1% leakage, $\frac{100-1}{100+1} = \frac{99}{101} = 0.98 =$ sin 78° . Thus, 1% leakage produces 2% error in the sine and 12 or 13% error in the birefringence. Nicol prisms instead of Polaroid will offset most of this error.

(3) Error in beam collimation due to irregularities in specimen, etc. This is a relatively small source of error, since it is approximately the same at both maximum and minimum position, and the optical system is shielded against scattering effects.

(4) Operating error (human error). For similar reasons to above, a small error near 90° will have much greater effect than same error away from 90° . Also, in this cateogory, is error due to not allowing enough time for needle to come to a full stop; parallax error, etc.

Some of these errors, if extensive enough, may cause relatively poor results even in regions away from 90° ; but, in all cases, the error in phase difference is much greater near the 90° -region, primarily due to the non-linear sine-angle relationship we have discussed above. Thus, the auxiliary plate (60°) becomes of major importance, since, even when lined up poorly with specimen, it will give a much more accurate reading than a direct reading from the 90° -region.

Accuracy Required.

While birefringence of only 5 millimicrons ($5/550 \times 360^\circ = 3.27^\circ$) per cm. may represent highly satisfactory (almost ideal) optical annealing, considerably higher path differences are acceptable for high-grade optical work. Nevertheless, the method described here should be adequate for testing optical blanks of such quality. Assuming the blank is at least 3 centimeters thick, we have a total birefringence of 15 millimicrons or about 10° of phase difference. We should then like to be able to measure to the nearest 10° of phase difference, which means that our error can't be greater than 4.9° or roughly 5° . Of course, we do not need this accuracy to enable our device to perform the functions for which it was designed, i.e., to discriminate quantitatively between acceptable and unacceptable optical blanks. However, with this accuracy, we know that our device satisfies the requirement "reasonably accurate" with considerable room to spare. A glance at the table on page 36 shows that our instrument does have accuracy well within 5° of error, or 1/72 of a wave-length, when the recommended procedure is used.

As indicated above, 10° of phase difference represents a path difference of about 15 millimicrons. Therefore, our permitted error of 5° represents about 7.5 millimicrons or 7.5×10^{-7} cm. of optical path difference. However, O.P.D. equals thickness multiplied by difference in index (by definition). Therefore, difference in index (in the planes of the fast and slow axes) is the O.P.D. divided by the thickness, or

$$= \frac{7.5X10^{-7}}{3} \cong 0.0000003$$

Finally, our permitted error in index-difference will be no greater than this seventh-place figure, and we should expect to measure to the nearest 0.0000006 of indicial-difference.

Conclusions:

The writer believes that this method has several important advantages over previous methods, especially in relation to large optical glass blanks. Some of these follows:

(1)-No lining-up of the specimen's fast and slow axis is necessary. This is especially important with medium and large optical blanks, where the rotation of the heavy, bulky flat about a given point would be very cumbersome if not prohibitive (for many such points). It is also important where very small amounts of birefringence exist and lining up is inherently difficult. Lining-up always provides an additional potential source of error and may require special skill and training on the part of the operator.

(2)-The instrument has a range of a full wave-length of path difference or $0°$ to $360°$ of phase difference. Thick optical blanks often show over a half-wave of total path difference (over $180°$ of phase difference). Other non-visual methods generally are limited to $90°$ of phase difference.

(3)-No extensive training or special skill is necessary for operation of this instrument in its basic form. Some special training is needed for the complete procedure from $0°$ to $360°$, but this is easily acquired. Other methods require laboratory skills such as microscopy, "lining up", photometric comparison, etc.

ADDENDA

Correction for Light-Leakage Through Polaroids:

A simple and accurate method of correcting the error produced by the approximately 0.7% leakage of plane-polarized light through the crossed position of each of our polaroid plates (1.4% total leakage) is to zero the ammeter while the collimated beam is incident on the same or similar polaroids in the crossed position. This will usually require removing the analyzer from the beam momentarily and inserting a plane polaroid plate just in front of (that is, nearer to the source than) the plane polaroid part of the circular polarizer with axes crossed to give "extinction". However, if an accurate quarter-wave plate is available, this could be inserted in the specimen position and the instruments' analyzer rotated to a minimum reading. This gives exactly the same results as crossed plane-polaroids. In either case the variac should be adjusted, after zeroing by shutter, to give a maximum reading of "MAX" ("60" on our scale) before zeroing through the crossed polaroids.

This method of zeroing provides us with "new" scale readings which are very nearly correct for all phase differences between 0° and 360°. The corrections for the small errors involved in this "new" scale can be derived by the method shown below.

Elliptic light emerging from the specimen can be treated mathematically as two plane-polarized beams, one maximum and one minimum in intensity, polarized perpendicularly to each other. The analyzer, in analyzing the minimum of these plane-polarized beams, is crossed with the maximum. Therefore, any leakage will be 1.4% of the maximum transmission, which is supposed to be extinguished at this instant, and this leakage, when zeroing by shutter only, will show on the ammeter-scale as a larger figure than the desired reading. Therefore, we must subtract it. Similarly, the leakage on analyzing the maximum beam will be approximately 1.4% of the minimum transmission, and this also will show as too large a figure on the ammeter's scale. Therefore, true phase difference at any minimum point, n, on the scale will be:

$$\text{ARC SIN}\ \frac{(60-\frac{1.4}{100}n)-(n-\frac{1.4}{100}\times 60)}{(60-\frac{1.4}{100}n)+(n-\frac{1.4}{100}\times 60)}$$

This formula is really all we need to correct our "old" scale and may be used to determine a table of corrections for it.

However, we have already decided to form our "new" scale by zeroing through crossed polaroids, that is, by subtracting our total leakage of 1.4% of 60 (our arbitrarily chosen maximum) from all points on the old scale. (This, of course, assumes our ammeter scale is linear). Therefore, "60" on our "new" scale would read "60 + 0.84" on our "old" scale (which we zero by closing the shutter.) (0.84 equal 1.4% of 60.) Similarly, any number, "n'", on our "new" scale would read "n' + 0.84" on our "old".

Substituting these quantities in our formula above, we find that the true birefringence for any minimum reading, n', on our "new" scale is:

$$\text{ARC SIN}\ \frac{[60.84-.014(n'+.084)]-[(n'+.084)-.014(60.84)]}{[60.84-.014(n'+.084)]+[(n'+.084)-.014(60.84)]}$$

$$\equiv \text{ARC SIN}\ \frac{60.84-1.014n'}{60.82+0.986n'}$$

We can now readily determine the true birefringence (in degrees of phase difference) for any points on our "new" scale and compare these results with the direct "new" scale readings. The difference will be the error for this reading on our "new" scale. This is worked out below for several points between 0° and 90° and a table of corrections for the entire birefringence scale is shown.

At 0° phase difference, as read after zeroing through crossed-polaroids, n' = 60. Applying our formula,

$$\sin\Delta\phi = \frac{60.84 - 1.014\,n'}{60.82 + 0.986\,n'} = \frac{60.84 - 1.014 \times 60}{60.82 + 0.986 \times 60} = \frac{0}{60.82 + .986 \times 60} = 0$$

and $\Delta\phi$ = 0° = true birefringence. Therefore, the correction here is zero.

At 30° phase difference, n' = 20. Substituting this value in our formula,

$$\sin\Delta\phi = \frac{60.84 - 1.014 \times 20}{60.82 + .986 \times 20} = \frac{40.56}{80.58} = 0.503$$

and $\Delta\phi$ = 30° 12' = true birefringence.

Therefore, the correction here is +12' of arc.

At 45° phase difference, n' = 10.3, and

$$\sin\Delta\phi = \frac{60.84 - 1.014 \times 10.3}{60.82 + .986 \times 10.3} = \frac{50.40}{70.98} = 0.710,$$

$\Delta\phi$ = 45° 14' = true birefringence. The correction is, therefore, +14' of arc.

At 60° phase difference, n' = 4.3, and

$$\sin\Delta\phi = \frac{60.84 - 1.014 \times 4.3}{60.82 + .986 \times 4.3} = \frac{56.48}{65.06} = 0.868$$

and $\Delta\phi$ = 60° 13' = true birefringence.

The correction is, therefore, + 13' of arc.

At 90° phase difference, n' = 0, and

$$\sin \Delta\phi = \frac{60.84 - 1.014 \times 0}{60.82 + .986 \times 0} = \frac{60.84}{60.82} = 1.000$$

and $\Delta\phi$ = 90° = true birefringence.

The correction is, therefore, zero.

Extending these findings to phase differences up to 360°, our table of corrections reads as follows:

Phase Difference (in degrees)	Correction (in minutes of arc)
0°	0'
30°0	+12'
45°	+14'
60°	+13'
90°	0'
120°	-13'
135°	-14'
150°	-12'
180°	0'
210°	+12'
225°	+14'
240°	+13'
270°	0'
300°	-13'
315°	-14'
330°	-12'
360°	0'

It becomes obvious, at once, that this correction table is unnecessary for the accuracy we desire and may be discarded.

The general conclusion from this discussion, therefore, is that we may obtain the accuracy we desire by zeroing the instrument through crossed-polaroids, as indicated above, and ignore any further scale corrections.

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Abstract of the Thesis: The Measurement of Birefringence in Optical Glass by Electronic Means.

This paper first presents a summary of several old and new methods of measuring birefringence, indicating their limitations in respect to large optical glass blanks, and then explains in considerable detail the writers' new radiation method, which is free of many of these limitations. The procedure and theory of this new method are given here briefly.

Procedure:

A collimated beam of monochromatic (yellow - green) light is passed through a circular polarizer and then through the specimen to be measured, that is, a large optical glass blank, which is held stationary. The beam next passes through a plane-polarized analyzer, which may be rotated to permit maximum and minimum transmissions of the elliptically polarized light emerging from the specimen. The intensity of this transmitted light is indicated on an ammeter by means of a sensitive photocell and amplifier.

Using a variac, the maximum intensity is adjusted to indicate a predetermined maximum position on the ammeter scale. The corresponding minimum position will then indicate the birefringence in terms of degrees of phase difference with the help of a special scale to be used in conjunction with the ammeter scale.

The circular light emerging from the circular polarizer enables the optical blank under test to be placed in any arbitrary azimuth in respect to its principal directions thus eliminating the necessity for lining up the heavy optical blank. The necessity for lining up is one of the chief objections to previous methods as far as their use with large optical blanks is concerned.

Theory:

When the circular light from the circular polarizer passes through the

specimen, the additional birefringence of the latter causes the beam to become elliptically polarized. Mathematical analysis of this elliptic light shows that the maximum and minimum intensities emerging from the rotatable plane analyzer determine by their ratio the birefringence of the specimen by the simple formula.

$$\sin \Delta\phi = \frac{A-B}{A+B} = \frac{\frac{A}{B}-1}{\frac{A}{B}+1}$$

where A is the maximum intensity, and B is the minimum, and $\Delta\phi$ is the birefringence of the specimen in terms of phase difference.

This device is to be used to measure birefringence in optical blanks between 0° and 360° of phase-difference. Therefore, to distinguish between the four possible phase-differences associated with each absolute sine value on the scale, advantage is taken of the fact that the major axis of our birefringence ellipse is orientated in the first 180° (0° - 180°) of specimen phase difference in a direction perpendicular to its orientation in the second 180° (180° - 360°) of specimen phase-difference. Thus, if we note that our maximum intensity position occurs with the analyzers indicator clockwise to the specimen's "fast" axis in the one group (0° - 180°) and anti-clockwise in the other group (180° - 360°), we can distinguish between these two groups. Also, by inserting a rotatable 60° - retardation plate in the beam's path, we can determine the sum of the retardations, and on subsequent subtraction of the known 60° - retardation, this will leave only one sensible answer, thus distinguishing between the (0° - 90°) group and the (90° - 180°) group, or their equivalent groups between 180° and 360°. By dividing the birefringence scale properly into upper and lower double groups of 90° span each, this procedure can be made reasonably simple. The "fast" axis of the optical blank at any point can be determined closely enough for the above purpose by the geometry of the round blank, (since principal directions are almost always radial and tangential), or else by previous qualitative examination.

Due to light-leakage through Polaroid plates at "extinction" positions, zeroing of the device may be accomplished by means of a shutter with only fair accuracy (relative inaccuracy around 90° of phase-difference). However, by zeroing the ammeter through crossed plane-Polaroids, we allow for most or all of this leakage at all phase-differences.

The principal advantage of the new method is the one previously suggested, namely, that we do not have to line up the heavy, bulky optical glass blank at each point to make the principal directions of the specimen coincide with those of the polarizer or analyzer. A second advantage over previous radiation methods is its full wave-length range. Another advantage is its simplicity of operation, at least for birefringence known to lie between 0° and 90° of phase-difference.